On the Impact of Probability Distributions of Stochastic Processes on Power System Dynamics

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***Abstract*—The paper originates from the observation that, when fitting stochastic processes based on data measurements, it is often possible to fit various probability distribution functions (PDFs) to the same data. This paper evaluates the impact of modeling stochastic processes with different PDFs on the dynamic behavior of the power system. The case study uses the well-known two-area system, which is modified to include stochastic processes through wind generation. Simulation results show that modeling stochastic processes such as wind speeds through different PDFs impacts the behavior of the power system differently, despite exhibiting similar statistical properties. The case study also shows that some PDFs worsen the effects of contingencies.**

***Index Terms*—Stochastic processes, probability distribution function, standard deviation, volatility, transient stability, wind generation.**

I. Introduction

A. Motivation

The increased penetration of renewable energy sources such as wind generation has posed an increased security risk on the power system operation. It has become essential to study the impact of wind generation on the power system security through dynamic studies. Traditionally, wind speed is characterized as a stationary stochastic process, and modeled as a stochastic differential equation (SDE) [1], [2]. An SDE has two features, namely the drift and diffusion term. The drift term defines the autocorrelation function and the evolution of the process in time. Whereas, the diffusion term defines the probability distribution function (PDF) of the process. This paper focuses on the evaluation of the impact of stationary stochastic processes defined through different PDF types on the dynamic behavior of the power systems.

B. Literature Review

Stochastic processes such as load power consumption, and wind and solar generation introduce volatility and uncertainty in power systems. The intensity of volatility and uncertainty is dependent on the PDF type with which an underlying stochastic process is modeled. Several PDF types can be fitted to a given measurement data using different available methods. The maximum likelihood method (MLE) is one of the most used methods to estimate the parameters of the fitting PDF type to the measurement data.

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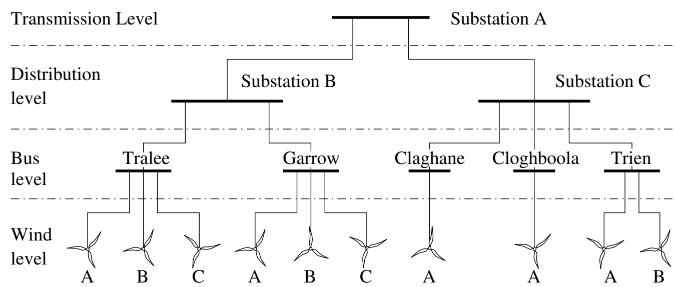

Fig. 1: Wind distribution network.

The limitation of MLE is that it does not provide any information on the quality of fit of the PDF to the data. The goodness of fit can be determined using different statistical tests. The outcome of a statistical test is dependent on the criteria chosen by test. Hence, these tests can yield multiple PDF types that best fit the data. In this situation, it is crucial to determine whether using different fitting PDF types for the data can yield modification of the dynamic behavior of the power system.

For simplicity but without lack of generality, in this paper, we focus on wind generation and assume it is the only source of volatility and uncertainty. Wind production is introduced into the power system through a distribution network of wind power plants (WPPs), shown in Fig. 1. This network was introduced in [3]. The distribution network in Fig. 1 depicts the WPPs organized in a tree-like structure, which is the real world scenario. Such WPPs present in geographical proximity are naturally connected in a tree-like structure, and due to the proximity exhibit correlated production characteristics.

A considerable amount of research work has been carried out in recent years on the modeling of correlated processes such as correlated load consumption and correlated wind speeds [4]–[7]. These works study the impact of correlated random processes on the dynamic of power system through differential-algebraic equations (DAEs). These studies include uncertainty into the set of DAEs at the point of initialization, i.e., the set of DAEs is randomly initialized by choosing the values from a given PDF. The DAEs once initialized randomly, stay deterministic for the rest of the simulation. This type of analysis is well suited for studying the effect of randomness in power systems.

In this paper, we are interested in studying uncertainty included in the power system in the course of time domain simulations. The impact of volatility and uncertainty in the presence of correlated stochastic processes on the power system dynamic behavior in the time-scale of voltage and rotor angle stability is studied by modeling the power system as a set of correlated stochastic differential-algebraic equations (SDAEs) [8], [9].

C. Contributions

This paper focuses on the impact of different PDF types of stochastic processes on the transient stability of power systems. In particular, the paper shows that the diffusion term of the stochastic processes when modeled with different PDF types, having similar statistical properties, may impact differently on the dynamic behavior of the power system.

D. Organization

The remainder of the paper is organized as follows. Section II provides a brief introduction of correlated stochastic differential-algebraic equations that are utilized to represent volatility in dynamic power system models. Section III introduces the correlated stochastic wind speeds. Section IV presents the simulation results obtained by simulating the detailed dynamic model of the two-area system, with the inclusion of correlated wind production, and analyzes the effect of correlated wind speeds with different PDF types on the dynamic behavior of the power system. Finally, Section V draws conclusions.

II. Modeling Correlated SDAEs

The impact of correlated stochastic processes on the dynamic behavior of power systems is defined through the set of n-dimensional correlated SDAEs, which are written as:

$$\begin{aligned} \dot{\boldsymbol{x}} &= \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{\eta})\,, \\ \boldsymbol{0} &= \boldsymbol{g}(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{\eta})\,, \end{aligned} \tag{1}$$

where $\boldsymbol{f} : \mathbb{R}^{l+m+n} \mapsto \mathbb{R}^m$, and $\boldsymbol{g} : \mathbb{R}^{l+m+n} \mapsto \mathbb{R}^l$ are the differential and algebraic equations; $\boldsymbol{x} \in \mathbb{R}^l$ is the vector of state variables; $\boldsymbol{y} \in \mathbb{R}^m$ is the vector of algebraic variables; and $\boldsymbol{\eta} \in \mathbb{R}^n$ represents correlated stochastic processes. These are modeled as a set of stochastic differential equations, as follows:

$$\dot{\boldsymbol{\eta}} = \boldsymbol{a}(\boldsymbol{\eta}) + \boldsymbol{b}(\boldsymbol{\eta}) \odot \boldsymbol{\zeta}\,, \tag{2}$$

where $\boldsymbol{a} : \mathbb{R}^n \mapsto \mathbb{R}^n$, and $\boldsymbol{b} : \mathbb{R}^n \mapsto \mathbb{R}^n$ are the vectors of so-called *drift*, and *diffusion* terms, respectively; $\odot$ represents the Hadamard product, i.e., the element-by-element product of two vectors; and $\boldsymbol{\zeta} \in \mathbb{R}^n$ is the vector of n-dimensional correlated *Gaussian white noise*, and is written as:

$$\boldsymbol{\zeta} = \mathbf{C}\,\boldsymbol{\xi}\,, \tag{3}$$

where $\boldsymbol{\xi} \in \mathbb{R}^n$ is a vector of uncorrelated Gaussian white noise. According to its formal definition, a Gaussian white noise process $\boldsymbol{\xi}(t)$ is the time derivative of the Wiener process $\boldsymbol{W}(t)$, as follows:

$$\boldsymbol{\xi}(t)\,dt = d\boldsymbol{W}(t)\,, \tag{4}$$

where $\boldsymbol{W} \in \mathbb{R}^n$ is a vector of uncorrelated standard Wiener processes i.e, the elements of $\boldsymbol{W}(t)$, say $W_i(t)$, $i = 1, \ldots, n$, are uncorrelated.

In (3), $\mathbf{C} \in \mathbb{R}^{n\times n}$ is a lower triangular matrix, and is calculated as Cholesky decomposition of the correlation matrix $\mathbf{R} \in \mathbb{R}^{n\times n}$ such that:

$$\mathbf{R} = \mathbf{C}\,\mathbf{C}^T\,. \tag{5}$$

The correlation matrix $\mathbf{R}$ represents the correlation among the stochastic processes defined in (2). Each element $r_{i,j}$ of $\mathbf{R}$ is written such that $r_{i,j} = \mathrm{corr}[dW_i(t), dW_j(t)]$, where $dW_i(t)$ is the increment of the i-th Wiener process. Note that $r_{i,j} = 1$ for $i = j$, since the correlation of a process with itself results in 1. Also note that for $\mathbf{R} = \mathbf{I}$, where $\mathbf{I}$ is the identity matrix, (2) results in uncorrelated stochastic processes.

The set of SDAE in (1)-(2) is highly non-linear, and is usually solved with numerical integration schemes. Equations (1)-(2) consist of two parts. The part involving the time derivative is deterministic and can be integrated using any usual integration scheme [10]. On the other hand, the part involving the integral w.r.t the Wiener process is a non-deterministic integral and is solved using Itô's calculus [11], [12].

III. Correlated Wind Speeds with given PDF

Correlated volatility on wind speeds is modeled through correlated SDEs, introduced in (2) and written as follows:

$$\dot{\boldsymbol{w}} = -\boldsymbol{\alpha} \odot (\boldsymbol{w} - \boldsymbol{\mu}) + \boldsymbol{b}(\boldsymbol{w}) \odot \boldsymbol{\zeta}\,, \tag{6}$$

where $\boldsymbol{w} \in \mathbb{R}^n$ represents the wind speeds; $\boldsymbol{\alpha} \in \mathbb{R}^n$, and $\boldsymbol{\mu} \in \mathbb{R}^n$ are the vectors of the autocorrelation coefficient, and the mean of the wind speeds, respectively; $\boldsymbol{\zeta}$ is the vector of correlated Gaussian white noise defined in (3); and $\boldsymbol{b}(\boldsymbol{w})$ are the diffusion terms. The expression of the diffusion term depends on the type of PDF utilized to model the wind speed. Different modeling techniques that define diffusion term are available in the literature. Two relevant examples are technique based on memoryless transformation [2] and stationary Fokker-Plank equation [13].

Equation (6) requires the correlation matrix $\mathbf{R}$, which contains information about the correlation between wind speeds $\boldsymbol{w}$. The elements of correlation matrix $\mathbf{R}$ are calculated from wind measurement data such that $r_{i,j} = \mathrm{corr}[dW_i(t), dW_j(t)]$, where $dW_i(t)$ is the increment of the i-th Wiener process. The i-th increment of the Wiener process for arbitrary time step Δt is written as [3]:

$$dW(t_i) = \frac{w(t_i) - w(t_{i-1})\,e^{-\alpha\Delta t} - \mu\,(1 - e^{-\alpha\Delta t})}{b\big(w(t_{i-1})\big)\sqrt{\frac{1-e^{-2\alpha\Delta t}}{2\alpha}}}\,. \tag{7}$$

IV. Case Study

This section illustrates the impact on the dynamic behavior of the power system of correlated wind speeds obtained through measurement data, and modeled with SDEs that reproduce different PDFs. For this reason, firstly, the wind measurement data in the time-scale of power system time domain simulations is presented in Section IV-A. Finally, Section IV-B studies the impact of different fitting PDF types of wind speeds on the transient behavior of the power system by simulating the well-known two-area system, modified to include wind power production. All simulation results presented in this section were obtain with the software tool Dome [14].

A. Data Analysis and PDF Fitting

The wind measurement data utilized in this case study were obtained from the Earth Observing Laboratory [15]. The real-world cumulative density function (RCDF) of the wind speed measurement data is shown in Fig. 2. The next step is to setup the parameters of the SDEs in (2) with a technique such as the one described in [2]. This also includes fitting a PDF to the measurement data.

We are interested in evaluating the impact of different PDFs on the overall dynamic response of the system. With this aim, we consider four PDFs fitting the data of Fig. 2, namely Gaussian, Beta, Gamma, and Weibull. The parameters of the four PDFs are determined through MLE. The cumulative density functions (CDFs) of the fitting PDF types under consideration are also illustrated in Fig. 2, along with the relative error between the CDFs of the PDF types and the RCDF of the wind speed measurement data. Figure 2 shows that there are minimal differences between the CDFs of the PDF types and the RCDF of the wind speed measurement data. At a first glance, thus, the four PDFs all fit reasonably well the data.

B. Dynamic Simulations

This section studies the impact of different PDF types of correlated stochastic wind speeds on the power system dynamic behavior. The power system utilized in this section is the well-known two-area system [16]. The two-area system contains in 11 Buses, 12 lines/transformers, and 4 synchronous generators. The synchronous generators are modeled via VI-order model, and are equipped with turbine governors, and automatic voltage regulator of IEEE Type-I. The two-area system is modified to include wind production through the wind distribution network shown in Fig. 1. This is done by connecting Substation A to Bus 9 of the two-area system.

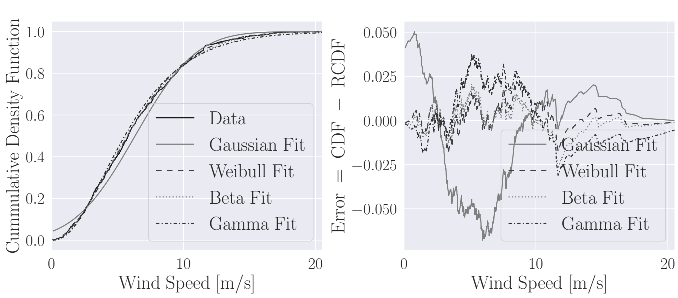

Fig. 2: Cumulative density function of measurement data and four PDF fits.

The correlation matrix $\mathbf{R}$ to simulate correlated wind speeds is taken from the wind measurement data provided in [3]. Two scenarios are considered, as follows:

- S1: fully uncorrelated wind speeds.
- S2: correlated wind speeds.

Each scenario simulates stochastic wind speeds through all the four PDF types discussed in section IV-A.

Both scenarios S1 and S2 are simulated using Monte Carlo time domain simulations, i.e., 1000 time domain simulations are performed per scenario. Correlated stochastic wind speeds are simulated using (6). The deterministic part of the SDAE in (1) is integrated using implicit trapezoidal integration scheme with a time step of $\Delta t = 0.01$ s. On the other hand, the integration of the diffusion term employs the Maruyama-Euler integration scheme with a step size of $h = 0.01$ s. The total simulation time for each trajectory is 100 s.

We first analyze the impact of simulating correlated stochastic wind speeds, through different PDFs, on the statistical properties of relevant quantities of the power system at the stationary conditions. The only difference in the four sets of simulation is the diffusion term in (6), which, as mentioned in the introduction, defines the PDF of the stochastic processes, in this case, wind speeds.

The drift term in (6), on the other hand, is assumed to be constant and same in all cases. The drift term defines the autocorrelation function of the wind speed. The autocorrelation functions of the total wind active power p_{wind} injected at Substation A into the power system is illustrated in Fig. 3. This figure shows that the autocorrelation functions of p_{wind} for all the scenarios are similar. This means that the drift terms of the wind speeds remain unaltered while simulating all the scenarios.

The impact of correlated stochastic wind speeds with different PDF types on the relevant power system quantities is quantified in Table I. Table I shows the standard deviation of the bus voltage magnitudes σ_v and the active power injections σ_p of the synchronous machines. Table I shows an increase in σ_v and σ_p from Gaussian to Gamma PDF in both scenarios. As expected, the values of σ_v and σ_p are higher in S2 as

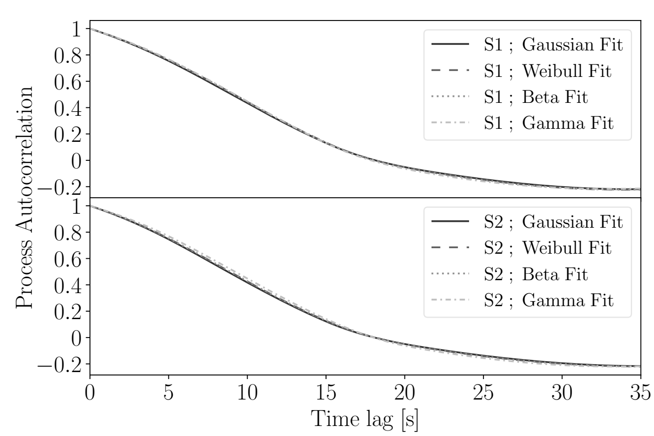

Fig. 3: Autocorrelation of p_{wind}.

TABLE I: Standard deviations (Std.) of power system quantities reached at stationary conditions.

Std. [pu]	Scenario S1							Scenario S2						
	Gaussian	Weibull	Error	Beta	Error	Gamma	Error	Gaussian	Weibull	Error	Beta	Error	Gamma	Error
$\sigma(v_{\text{Bus 08}})$	0.0042	0.0047	11.90	0.0048	14.29	0.0049	16.67	0.0087	0.0091	4.60	0.0092	5.75	0.0095	9.20
$\sigma(v_{\text{Bus 09}})$	0.0028	0.0031	10.71	0.0032	14.29	0.0033	17.86	0.0058	0.0061	5.17	0.0062	6.90	0.0064	10.34
$\sigma(p_{\text{G1}})$	0.0059	0.0067	14.80	0.0069	16.86	0.0075	27.32	0.0136	0.0136	4.08	0.0138	5.97	0.0148	13.10
$\sigma(p_{\text{G2}})$	0.0058	0.0067	14.77	0.0068	16.84	0.0074	27.21	0.0135	0.0135	4.07	0.0138	5.96	0.0147	13.08
$\sigma(p_{\text{G3}})$	0.0058	0.0067	15.14	0.0068	17.08	0.0075	29.32	0.0134	0.0134	4.00	0.0137	5.89	0.0146	13.32
$\sigma(p_{\text{G4}})$	0.0058	0.0066	15.06	0.0068	17.02	0.0074	28.98	0.0133	0.0133	3.98	0.0136	5.87	0.0145	13.27

Error: Normalized Error calculated in % with Gaussian PDF as base.

compared to S1. This happens because S2 considers correlated wind speeds [3].

The results in Table I are noteworthy because the only parameter that varies during the time domain simulations is the PDF of the wind speed. The changes in the statistical properties of the power system quantities based solely on PDF types of wind speeds are counter intuitive. One would expect to see no differences in the statistical properties of any of the power system quantity based on different PDF types of wind speeds. Especially when the differences between CDFs of the fitting PDF types and the RCDF of data are small.

Further insights on the effect of the PDFs on the dynamic response of the system can be obtained with a frequency domain analysis. With this aim, we first evaluate the amplitude of the oscillations induced in the inter-area electromechanical oscillatory mode of the power system [17]. The inter-area mode of the modified two-area system is calculated as, eigenvalue -0.075167 ± 3.540781, and frequency 0.563 [Hz]. Then we evaluate the frequency spectrum of p_{wind} for the two scenarios and four PDFs, which is illustrated in Fig. 4. Results show that the amplitude of the frequencies in p_{wind} is dependent on the PDF types of the underlying wind speeds. In the two scenarios, the amplitudes of frequencies in case of Gaussian PDF are the lowest whereas Gamma PDF shows the highest amplitude. On the other hand, the amplitudes of frequencies for Weibull and Beta PDF are remarkably similar in the whole frequency spectrum.

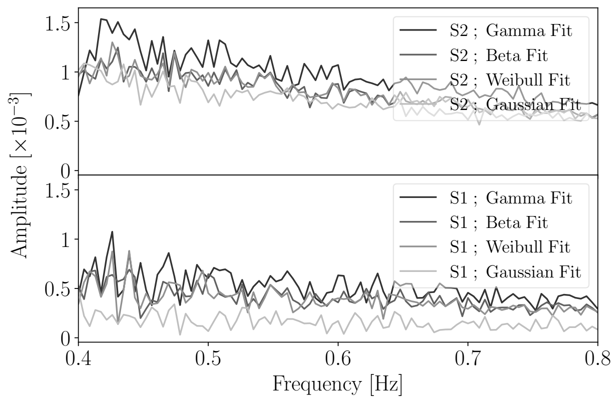

Fig. 4: Frequency spectrum of p_{wind}.

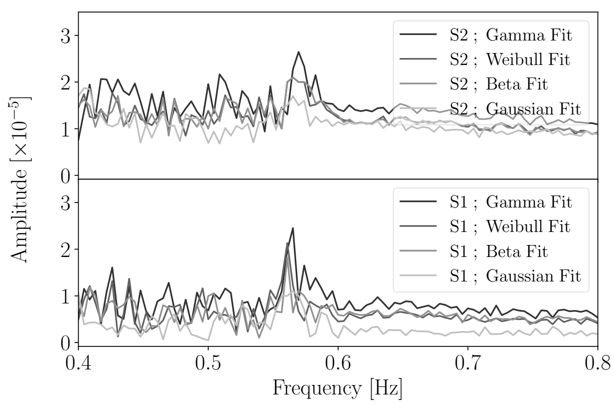

Fig. 5: Frequency spectrum of $v_{\text{Bus 08}}$.

The amplitude of oscillations induced in the inter-area oscillatory mode is shown in Fig. 5, which illustrates the frequency spectrum of voltage magnitude at Bus 8 $v_{\text{Bus 08}}$. Figure 5 shows highest amplitude of oscillations in the inter-area oscillatory mode for Gamma PDF with the lowest being the Gaussian PDF for both scenarios. These variations in the amplitudes of the oscillations for different PDFs lead to variations in the statistical properties of the quantities of the power system as seen in Table I.

Next, we consider the impact of the PDFs on the behavior of the system after the occurrence of a contingency. This consists in the trip of the line connecting buses 8 and 9 at time $t = 30$ s. The trajectories of $v_{\text{Bus 08}}$ obtained as a result of simulating correlated wind speeds, through different PDF types, are illustrated in Figs. 6 and 7 for scenarios S1 and S2, respectively. In both scenarios, the standard deviation of $v_{\text{Bus 08}}$ is the lowest for the Gaussian and the highest for the Gamma PDF. Table I shows the values of $\sigma_{v_{\text{Bus 08}}}$ before the contingency. Figure 6 shows that when the wind speeds are not correlated (S1), the trajectories of $v_{\text{Bus 08}}$ remain above the minimum voltage limit for the entire simulation. On the other hand, for S2, namely for the scenario of correlated wind speeds, a considerable number of trajectories of $v_{\text{Bus 08}}$ violate the minimum voltage limit, as seen in Fig. 7. The number of trajectories of $v_{\text{Bus 08}}$ that go below the minimum voltage limit at least once in the period of 30 s $< t <$ 35 s is shown in Table II.

The results shown in Figs. 6 and 7, and Table II agree with the discussion presented above in this Section. In both scenarios, the Gamma PDF leads to the worst dynamic behavior whereas the Gaussian PDF to the best. The Weibull and Beta PDF remain close to each other, which has to be expected as the differences in the CDFs of Weibull and Beta PDF vs RCDF as well as in their frequency spectrum are negligible.

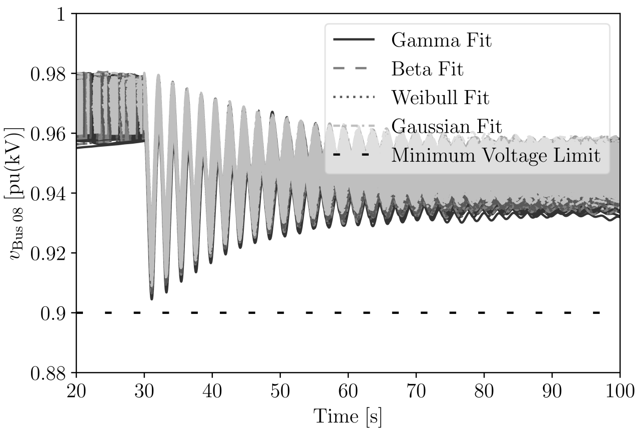

Fig. 6: Voltage profile at bus 8 for scenario S1.

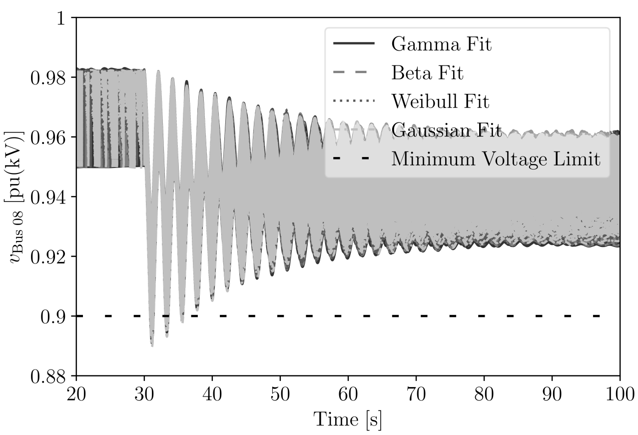

Fig. 7: Voltage profile at bus 8 for scenario S2, where MVL is Minimum Voltage Limit.

TABLE II: Percentage of trajectories below voltage limit for scenario S2.

PDF	Trajectories with under-voltages
Gaussian	27 (2.7 %)
Weibull	38 (3.8 %)
Beta	44 (4.4 %)
Gamma	70 (7.0 %)

These results, while being non-intuitive, can be understood by analyzing the oscillations induced in the power system by the wind speeds following different PDFs.

V. Conclusions

This paper studies the impact of modeling correlated stationary stochastic processes with different PDF types on the dynamic behavior of power systems. The case study shows that stochastic processes modeled with different PDFs might have different effects on the statistical properties of the power system quantities as compared to other PDF types. This happens even though the PDFs exhibit similar statistical properties and all other parameters of the stochastic processes and the rest of the system are the same.

The conclusions that can be drawn in this case study do not allow to conclude that the Gamma PDF *always* leads to the worst dynamic response, nor that processes with different PDFs always cause different dynamic impacts. The effect of the PDF depends on the autocorrelation of the stochastic processes, their locations in the network and on the oscillatory modes of the system.

Assessing the effect of a specific PDF on the dynamic behavior of the system is a not a straightforward task to solve as both system equations and the diffusion terms of the processes are nonlinear. In this paper, we have relied on time domain simulations. However, these are cumbersome as they require a Monte Carlo analysis for each considered PDF. We believe that the analysis of the spectra of random processes characterized by different although very similar PDFs is a promising alternative approach. Even if the PDFs are similar, in fact, the spectra are slightly different and so might be their impact on the dynamic of the system. How to exactly quantify this impact is currently an open question for us and we are working on finding an answer. In particular, future work will focus on developing techniques, e.g. based on frequency analysis, that allow identifying the worst performing PDF without resorting to time domain simulations.

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