Continuous-Time ARMA Models for Data-based Wind Speed Models

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Abstract—The focus of this paper is on the dynamic analysis of power systems including wind generation. A continuoustime implementation of the well-known Autoregressive Moving Average processes is proposed. This model is based on measured data and built through a four-step procedure that retains both the probability distribution and the autocorrelation of the actual wind speed. The effect of the data sampling rate on the stochastic properties of the wind speed is evaluated through the comparison of real-world hourly and minutely data. Finally, the proposed model is included in a benchmark system and its transient behavior is discussed.

Index Terms—Continuous-Time Autoregressive Moving Average (CARMA), Stochastic Differential Equation (SDE),Wind speed modeling.

I. INTRODUCTION

A. Motivation

Wind power generation has increased significantly in recent years and has become a prominent part of the energy production portfolio [1]. The stochastic nature of wind introduces uncertainty and volatility into power grids. The impact of this volatility on the dynamic behavior of power systems has not been thoroughly investigated and remains a relevant research question. TO this aim, accurate stochastic models are required so that the wind speed fluctuations can be captured in simulation. This paper studies the effect of the data sampling rate on wind speed models intended for generating synthetic data for dynamic analysis of power systems.

B. Literature Review

From a statistical point of view, wind speed can be characterized by its probability distribution and autocorrelation. The best fit for the autocorrelation and probability distribution of wind speed is dependent on both the location and the sampling time [2], [3]. An accurate definition for the autocorrelation is important as it specifies by how much the wind speed varies within a certain time frame. An inaccurate definition of either the probability distribution or the autocorrelation of the wind speed model results in implausible wind generation meaning the resulting power system simulations are unrealistic. Traditional wind speed models used for time-domain simulation of power systems fail to capture the probability distribution, the autocorrelation or both [3]–[5]. For example, the fourcomponent composite model proposed in [4] is designed to model the spatial effect of wind behavior on the wind-turbine and includes gusts, rapid changes and background noise. This model fails to capture any statistical property of the wind. Other wind speed models available in the literature capture the probability distribution but neglect the autocorrelation [5]. These models cannot guarantee that the variations of the generated wind speed are realistic. Thus, simulations using these models do not properly represent the real-world system.

In [2], a systematic procedure to build a Stochastic Differential Equation (SDE) with an exponentially decaying autocorrelation is presented. Such a procedure provides a good fit to hourly wind speed data where the daily fluctuations are not visible [6]. However, hourly data fail to capture *fast* wind variations that, as this paper demonstrates, have to be taken into account for accurate dynamic analysis of power systems. Such short-term wind speed variations, e.g. turbulence and gusts, typically occur within a 10 minute time frame and result in the autocorrelation initially decreasing rapidly before settling to the same slope as the hourly data. The resulting autocorrelation is not exponentially decaying. Thus, the models presented in [2] cannot be used.

Another category of wind speed models are based on Autoregressive Moving Average (ARMA) models. These models have been widely utilized for wind speed modeling [7], forecasting [1], [8] and to analyze the impact of wind integration on power system reliability and long-term planning [9]. They are well-established and offer comprehensive tools to fit the model to data and reproduce both the distribution and the autocorrelation. However, ARMA models are discrete and have a fixed time step that must match the sampling interval of the available data. Therefore, ARMA models are not suitable for dynamic analysis of power systems as they typically have a smaller time step than the available data and/or require a varying time step.

C. Contributions

The focus of this research is to propose an approach that enables the utilization of ARMA models in the continuoustime domain. This facilitates their use in time-domain simulations of power systems for dynamic analysis. The proposed approach is based on the observation that if an ARMA

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model is stationary then it has an equivalent Stochastic Differential Equation (SDE), termed a Continuous-Time ARMA (CARMA) [10].

The specific contributions of this paper are twofold:

- To present a method to construct continuous-time wind speed models, for the time-domain simulation of power systems, which produce the probability distribution and the autocorrelation of wind speed data that are equivalent to ARMA models.
- To determine if wind speed models based on hourly data are adequate for dynamic analysis of power systems. This is achieved by comparing wind speed models based on hourly and minutely data and implementing the models in a benchmark power system.

D. Organization

The reminder of this paper is organized as follows. The steps to construct SDE-based CARMA wind speed models are presented in Section II. In Section III, wind speed models based on data with hourly mean values are constructed using the proposed CARMA procedure and compared to wind speed models constructed using the Fokker-Planck approach described in [2]. Section IV outlines the difference between hourly and minutely wind speed data and presents CARMA models that accurately model minutely-sampled wind speeds. In Section V, the impact of including the proposed CARMA wind speed models in a benchmark power system model is studied. Finally, Section VI draws conclusions and outlines possible areas of future research.

II. CONSTRUCTION OF CARMA MODELS

This section presents a procedure to obtain a continuoustime ARMA model from wind speed measurements. The proposed procedure consists of four steps, as shown in Fig. 1. Each step is detailed below.

Step 1: Memoryless transformation

Typically, wind speed data do not have a Gaussian distribution. It is commonly shifted to the left and tails heavily to the right. The Weibull distribution is an example of this and is the most widely-used probability distribution for wind speed. However, ARMA and CARMA models require the data to be normally distributed. Therefore, the wind speed data must be fitted to the Gaussian distribution. This is achieved using a memoryless transformation.

The memoryless transformation fits non-normally distributed data to the Gaussian distribution while retaining its original stochastic properties. This is achieved by applying the inverse Gaussian Cumulative Distribution Function (CDF) to the CDF of the wind speed data, as follows:

$$y(t) = \Phi^{-1}(F(X(t))), \qquad (1)$$

where Φ^{-1} is the inverse CDF of the Gaussian distribution and F is the CDF of the probability distribution of the wind speed [11]. The result is a time series that is normally distributed and can be used to build an ARMA model.

Step 2: ARMA modeling

ARMA models can model and forecast future wind speed variations based on historical data. It is possible, for a large class of autocorrelation functions, $\gamma(\cdot)$, to find an ARMA model with the autocorrelation function, $\gamma_X(\cdot)$, such that $\gamma(\cdot)$ is well approximated by $\gamma_X(\cdot)$. These models have wellestablished estimation techniques that simplify the building of custom wind speed models for each location based on available measurements [12].

ARMA models can be divided into two components, namely, Autoregressive (AR) and Moving Average (MA):

- *Autoregressive*: relates the current value of the wind speed to past values.
- *Moving Average*: relates the current value of the wind speed to past error values.

The ARMA(p, q) model is given by

$$X_{t} = \underbrace{\sum_{i=1}^{p} \phi_{i} X_{t-i}}_{AR} + \underbrace{\sum_{i=1}^{q} \theta_{i} \varepsilon_{t-i}}_{MA} + \varepsilon_{t} , \qquad (2)$$

where ε_t is white noise with a standard deviation σ_a , ϕ_i are the autoregressive parameters, θ_i are the moving average parameters and both ϕ_p and θ_q are non-zero.

ARMA models of second order or higher have been widely used to model wind speed [7]–[9]. The ARMA(2,1) model is a special case as it is the lowest order ARMA model that captures the statistical properties of wind speed. It can be written as

$$\phi(B)X_t = \theta(B)\varepsilon_t , \qquad (3)$$

where B is the backward operator such that $BX_t = X_{t-1}$ and

$$\phi(z) = 1 - \phi_1 z - \phi_2 z^2 \tag{4}$$

$$P(z) = 1 + \theta_1 z . \tag{5}$$

Several well-known methods are available to estimate ARMA parameters directly from data such as the Least Squares method, the Method of Moments and the Maximum Likelihood method [13]. In this research, the Maximum Likelihood method is used. The method finds the parameter values of the ARMA model which maximizes the Likelihood Function of the sampled data. The Likelihood Function is based on the Gaussian CDF of the sampled data. The estimated ARMA parameters are used to find the equivalent CARMA parameters.

Step 3: CARMA modeling

Wind speed is a continuous-time process. However, wind speed measurements are sampled and the resulting data is discrete. For this reason, ARMA models are commonly used in practice.

CARMA models are the continuous-time counterparts of the discrete-time ARMA models. While ARMA models are constrained by the fixed time step of the sampled data, CARMA models enable the utilization of any time step,



Figure 1. The four steps to construct the proposed SDE-based CARMA wind speed models.

including variable ones. Hence, CARMA models can be used to interpolate between sampling points.

A CARMA(p, q) model denoted by x(t) is a SDE of the form

$$\frac{d^{p}x}{dt^{p}} + c_{p-1}\frac{d^{p-1}x}{dt^{p-1}} + \dots + c_{1}\frac{dx}{dt} + c_{0}(x(t) - \mu)$$
$$= b_{0}dW(t) + b_{1}\frac{dz}{dt} + \dots + b_{q}\frac{d^{q}z}{dt^{q}}, \qquad (6)$$

where W(t) is the standard Weiner process, c_i are the autoregressive coefficients, b_i are the moving average coefficients and both c_i and b_i are real and $b_q \neq 0$ [14]–[16].

Generally, a stationary CARMA(p, q) model sampled regularly can be written as a ARMA(p, p-1) model with q < p[10]. The simplest example is the Ornstein-Uhlenbeck process, i.e., CARMA(1,0):

$$dX_t + c_0 X_t = dW(t) , \qquad (7)$$

which is equivalent to an ARMA(1,0) model viewed with a fixed time step h:

$$X_t = \exp(-c_0 h) X_{t-1} + \varepsilon_t .$$
(8)

Note, the autoregressive parameter of the discrete-time ARMA, $\phi_1 = \exp(-c_0 h)$, cannot be negative.

The CARMA(2,1) model used in the remainder of this paper and can be written as

$$c(D)X(t) = b(D)dW(t) , \qquad (9)$$

where D is the differential operator and

$$c(z) = z^2 + c_1 z + c_0 \tag{10}$$

$$b(z) = b_0 + b_1 z . (11)$$

An equivalent discrete-time ARMA(2,1) model can be found if the CARMA(2,1) model is stationary. The CARMA(2,1) model is stationary if the real parts of the roots of (10), α_1 and α_2 , are negative. The autoregressive parameters of the continuous-time model, c_1 and c_0 , can be directly connected to the autoregressive parameters of the discrete-time model, ϕ_1 and ϕ_2 , using the z-transformation.

$$\phi_1 = e^{\alpha_1 h} + e^{\alpha_2 h} \tag{12}$$

$$\phi_2 = -e^{(\alpha_1 + \alpha_2)h} \,. \tag{13}$$

The theoretical auto-covariance function of a discrete-time ARMA(2,1) model is defined as

$$\gamma_{\text{ARMA}}(k) = \begin{cases} \phi_1 \gamma(1) + \phi_2 \gamma(2) + \theta_1 (\phi_1 + \theta_1) \sigma_a^2 + \sigma_a^2 & \text{if } k = 0\\ \\ \phi_1 \gamma(0) + \phi_2 \gamma(1) + \theta_1 \sigma_a^2 & \text{if } k = 1\\ \\ \phi_1 \gamma(k-1) + \phi_2 \gamma(k-2) & \text{if } k > 1 \\ \end{cases},$$
(14)

where k is the time lag and $\gamma = \gamma_{ARMA}$. The theoretical auto-covariance function of a CARMA(2,1) model is

$$\gamma_{\text{CARMA}}(h) = e^{\alpha_1 h} \frac{b(\alpha_1)b(-\alpha_1)}{c'(\alpha_1)c(\alpha_1)} + e^{\alpha_2 h} \frac{b(\alpha_2)b(-\alpha_2)}{c'(\alpha_2)c(\alpha_2)} .$$
(15)

The moving average parameter of the continuous-time model, b_1 , is set so that the auto-covariance of the discretetime model, γ_{ARMA} , is equal to the auto-covariance of the continuous-time model, γ_{CARMA} . It is important to note that the work in this paper can be extended to map the parameters from any CARMA(p, q) model, where p > q, to find the equivalent ARMA(p, p-1) parameters [14].

The resulting wind speed CARMA model is normally distributed. Hence, its true probability distribution has to be imposed. This is achieved using the inverse memoryless transformation.

Step 4: Inverse memoryless transformation

The inverse of the memoryless transformation in *Step 1* is used to impose the true probability distribution of the data. The inverse CDF of the wind speed data is applied to the Gaussian CDF of the CARMA model

$$y(t) = F^{-1}(\Phi(X(t))) .$$
(16)

This obtains the desired probability distribution of the wind speed data [11]. The memoryless transformation and its inverse enable the use of ARMA and CARMA models to model data with any probability distribution that has a defined CDF and inverse CDF.

III. SDE-BASED HOURLY WIND SPEED MODELS

In this section, hourly-mean wind-speed measurements collected in two locations in Ireland are considered. The construction method presented in Section II and the Fokker-Planck method presented in [2] are used to build models based on the hourly wind speed data. The results from these two methods are compared.

A. Wind speed data

The wind speed datasets used throughout this research were provided by Met Éireann for two locations in Ireland [17]. Each dataset consists of three years of hourly wind speed measurements.

- Dataset 1: Moore Park in county Cork that is located inland in the south of Ireland.
- **Dataset 2**: Valentia Observatory in county Kerry that is located on the south-west coast of Ireland.

The Probability Density Functions (PDFs) presented in [2] are fitted to the datasets and the Kolmogrov-Smirnov test is used to determine which of the PDFs best fits the data. The resulting PDFs that best fit the datasets are presented in Table I. Figure 2 shows the fitted PDF of the two datasets. The PDF of Dataset 1 has a high peak to the left which indicates that there is high probability of low wind speeds. On the other hand, the measured data gathered at the Valentia Observatory have a flatter PDF, meaning that high wind speeds are more common.



Figure 2. The fitted PDFs for the wind speed datasets in Section III-A.

The autocorrelation of the measured data is shown in Fig. 3. The oscillations have peaks separated by 24 hours owing to similarities in daily wind speeds. These oscillations have a high amplitude in the inland wind speed data but are almost undetectable in the autocorrelation of the coastal wind speed data. If the autocorrelation is not properly defined the wind variations may be incorrectly modeled. The models presented in this paper aim to match the autocorrelation over the first 12 hours and hence neglect daily variations. In cases where daily fluctuations cannot be neglected, the well-established seasonal modeling ARMA addition to can be used [18].

B. Simulation results

The wind speed is modeled to fit the data in Section III-A using the following two methods:

- Method I: The CARMA-based SDE construction method presented in Section II.
- **Method II**: The construction method presented in [2]. This method utilizes the stationary Fokker-Planck equation to impose the desired probability distribution and the



Figure 3. The autocorrelation of the wind speed datasets in Section III-A.

Regression Theorem to impose an exponentially decaying autocorrelation.

The models are simulated 1,000 times, producing three years of synthetic hourly data. The estimated PDF parameters, the mean and the standard deviation are computed for each process using both methods. The average values of the 1,000 generated processes are shown in Table I. The differences between the estimated parameters and the parameters of the data are included as a percentage in Table I. The estimates that are closer to the parameters of the data are highlighted.

The autocorrelation is computed for each of the 1,000 processes produced using both methods. The average autocorrelation for each method is computed and the difference between the computed autocorrelation and the autocorrelation of the datasets is found. The difference in the autocorrelation for both datasets are shown in Figs. 4 and 5. Method I provides a better fit to the autocorrelation of Dataset 1 as it has a smaller error than Method II. On the other hand, Method II provides a better fit for Dataset 2 as it has low daily variations in the wind speed.



Figure 4. A comparison of the autocorrelation of Dataset 1 and the average autocorrelation of the 1,000 processes generated using Methods I and II.

Both methods are able to reproduce the distribution and the autocorrelation of the datasets with a maximum error of 7%. The results indicate that both methods can be used for modeling hourly wind speed data.

Note, if the hourly wind speed data for a certain location cannot be modeled using a CARMA(2,1) model, a higher

TABLE I

Probability density function parameters $(\lambda_1, \lambda_2, \lambda_3)$, mean (μ) and standard deviation (σ) of the datasets in Section III-A compared to the results from 1,000 simulations of the wind speed models generated using Methods I and II.

				Method I		Method II	
Dataset	Probability Distribution	Parameters	Data	Estimate	Difference [%]	Estimate	Difference [%]
1. Moore Park, Cork	3-parameter Beta	λ_1	2.1686	2.2735	4.8361	2.1617	0.3147
		λ_2	14.0290	14.6901	4.7123	13.3632	4.7463
		λ_3	22.09827	22.3398	1.0931	21.3330	3.4631
		μ	2.9449	2.9379	0.2361	2.9755	1.0392
		σ	1.8283	1.7569	3.9026	1.8155	0.6997
2. Valentia Observatory,	3-parameter Gamma	λ_1	0.5533	0.5154	6.8630	0.5615	1.4768
Kerry		λ_2	8.5203	8.8969	4.4201	8.5299	0.1124
		λ_3	2.5320	2.5816	1.9605	2.5462	0.5618
		μ	5.3769	5.5016	2.3199	5.5276	2.8041
		σ	3.2677	3.3167	1.5019	3.2147	1.6215



Figure 5. A comparison of the autocorrelation of Dataset 2 and the average autocorrelation of the 1,000 processes generated using Methods I and II.

order CARMA model is required.

Figure 6 shows a comparison between hourly and minutely wind speed data. The hourly sampled data fail to capture the faster variations of the wind speed occurring within a 10 minute time frame. These fast wind speed variations cannot be neglected as they are required for the transient stability analysis of power systems. Note, these faster fluctuations are filtered due to the damping effect of the wind turbine blades. Further analysis on the relationship between the damping effect and the wind speed data sampling rate is presented in [19].



Figure 6. A comparison of wind speed data gathered hourly and minutely in the Valentia Observatory, Kerry, Ireland [17].

IV. SDE-BASED MINUTELY WIND SPEED MODELS

This section compares hourly and minutely wind speed data and demonstrates that a construction method that can produce wind speed models with non-exponentially decaying autocorrelation is necessary. It is established that the CARMA construction approach presented in Section II can accurately capture the properties of minutely wind speed data.

A. The wind speed data

The wind speed measurements used in this section were gathered at Valentia Observatory in Kerry. The data consist of minutely and hourly data over a one month period.

The same procedure as in Section III-A is used to find the best fitting PDF. The resulting PDFs for the hourly and minutely data are shown in Fig. 7. The lower wind speeds are slightly more prevalent in the hourly data while the minutely data have a flatter PDF. An hourly sampling is sufficient to capture the PDF of the wind speed as the difference between the two PDFs is minimal.



Figure 7. A comparison of the PDFs of the wind speed data gathered hourly and minutely in the Valentia Observatory, Kerry, Ireland [17]. The CARMA model PDF is the average of 1,000 simulations of the wind speed model based on the minutely data.

Figure 6 demonstrates that hourly data do not capture fast variations in the wind speed that are visible in the minutely data. These short term variations in the wind speed result in a fast drop in the autocorrelation within the first few minutes. After the first 20 minutes, the autocorrelation settles to the same slope as the autocorrelation of the hourly data (see Fig. 8). Models based on hourly data generate processes with lower fluctuations than those that are observable in the actual wind speed.



Figure 8. A comparison of the autocorrelation of the wind speed data gathered hourly and minutely in the Valentia Observatory in Kerry, Ireland [17].

B. Simulation results

The wind speed is modeled to fit the data in Section IV-A using the CARMA construction method presented in Section II. The model is simulated 1,000 times producing one month of synthetic minutely data.

The resulting PDF for the 1,000 simulations is shown in Fig. 7. The CARMA-based model has a PDF where low wind speeds are marginally more likely.

The average autocorrelation of the 1,000 generated processes and the autocorrelation of the minutely data are shown in Fig. 9. The CARMA(2,1) model captures the autocorrelation of the minutely data for lags of up to 20 minutes. Its autocorrelation diverges from the autocorrelation of the data over longer time periods. This is sufficient if the wind speed model is intended for angle and voltage stability analysis of power systems. A higher order CARMA model is required to effectively capture the autocorrelation for higher lags.



Figure 9. The average autocorrelation of the 1,000 CARMA processes modeled based on minutely data gathered at Valentia Observatory, Kerry, compared to the actual autocorrelation of the data.

V. SENSITIVITY ANALYSIS OF A POWER SYSTEM MODEL INCLUDING SDE-BASED WIND SPEED MODELS

The power system used to test the CARMA wind speed models is a modified version of the WSCC 9-bus system shown in Fig. 10. This system consists of 3 synchronous machines, 3 two-winding transformers, 5 transmission lines and 3 loads. The system also includes primary voltage regulators (AVRs), turbine governors and an AGC. Further details on the system can be found in [20]. The following modifications have been made to the system:

- The capacity of the synchronous generator at Bus 2 is reduced by 20 MW.
- A wind power plant is connected to the system at Bus 7 through a two-winding transformer with the power capacity 20 MW. The wind turbine model used is a Double-Fed Induction Generator (DFIG) with MPPT, pitch angle and voltage controls.



Figure 10. Modified WSCC 9-bus test system.

The CARMA wind speed models for the Valentia Observatory in Kerry for hourly data, presented in Section III, and for minutely data, presented in Section IV, are used. The power system model is simulated 1,000 times for 100 s. The initial wind speed for all simulations is set so the wind power plant generates 20 MW.

Figure 11 shows that the wind speed changes more over the 100 s for the model based on the minutely data. The effect of this difference is visible in the active power generated by the wind power plant and at all the buses of the system.



Figure 11. 1,000 generated wind speed processes based on hourly and minutely data fed to the wind power plant in the test system.

Figure 12 shows the change in the voltage at Bus 7 resulting from the fluctuations of the wind speed. The wind power plant reaches its lower limit when minutely based processes are employed. This can be observed in Fig. 12 and explains the hard lower limit of the voltage fluctuations.



Figure 12. Voltage trajectories at Bus 7 of the test system for the 1,000 simulations using the hourly and minutely based wind speed models.

The difference between using minutely and hourly data to model wind speed is clear in the time-domain simulation of the power system. These results highlight the importance of the data sampling rate when building wind speed models for dynamic analysis of power systems.

VI. CONCLUSIONS

In this paper, a four step procedure to produce CARMA wind speed models from data is presented. These models are designed to capture both the PDF and the autocorrelation of the wind speed data and are intended for dynamic analysis of power systems.

The presented CARMA construction method is compared to a Fokker-Planck based method that imposes an exponentially decaying autocorrelation. Both methods are able to reproduce the PDF and autocorrelation of the two hourly wind speed datasets used.

The hourly wind speed data do not capture the variations of the wind that occur within a 10 minute time frame. These fast variations in the wind result in the autocorrelation of the minutely wind speed being non-exponentially decaying. The paper shows that the CARMA construction method can be used to build wind speed models that capture the PDF and autocorrelation of minutely data.

Finally, the proposed wind speed models based on hourly and minutely data are included in a test power system. It is shown that hourly data are not adequate for building wind speed models for dynamic analysis of power systems.

Future work will focus on the effect of detailed wind speed models on the transient behavior of real-world power systems. High order CARMA models will be considered to create these detailed wind speed models.

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