On the Impact of Topology on Power System Transient and Frequency Stability

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***Abstract*—This paper presents a statistical study on the impact of topology of the transmission system on the transient and frequency stability of power systems. We consider four topologies: random graph; small-world graph; nested small-world graph; and lattice graph. These graphs are utilized to generate hundreds of synthetic networks with given topology and adequately generated static and dynamic data. Then, each network is simulated based on a Monte Carlo method and considering random contingencies and different levels of inertia. The case study allows drawing some remarkable conclusions on the correlation between power system stability and topology.**

***Index Terms*—Network topology, transient stability, frequency stability, synthetic networks, Monte Carlo simulation.**

I. Introduction

The availability of the data of real-world large power systems is scarce and often subject to strict confidentiality agreements. This has significantly limited data sharing and, hence, the ability to reproduce results. Even if data of real-world systems were public domain, however, these would not constitute a valid statistical population sample, as there are not actually many large interconnected systems in the world. A Monte Carlo analysis of the impact of topology on interconnected systems is thus only possible through the generation of synthetic networks.

A few algorithms are available in the literature. In [1] and [2], a tree-structured power model is used to study power grid robustness and cascading failure blackouts, in [3] a ring-structured power grid is developed to study the pattern and speed of contingency. An algorithm to generate power system based on small-world graphs is presented in [4]. A mechanism of chain failure steps in a large-scale grid is studied in [5] also based on the small-world graph model. In [6], a random topology power grid model, called nested small-world, based on a comprehensive study of the topology and electrical properties of realistic grids is proposed. In [7] synthetic power networks are created using a clustering technique to locate the substations according to public U.S. vensus database records. A set of validation criteria is used to add a network of transmission lines at multiple voltage levels so that the built synthetic networks can match the statistical characteristics found in actual power networks.

The works above are aimed at generating steady-state data and are being utilized to generate realistic synthetic networks for power flow and optimal power flow studies [7]. The generation of synthetic dynamic data has not been treated systematically so far. However, it is relatively easy to address this task, as typical data for synchronous machines and their regulators can be found in the literature, an pu values of machines and regulators of a given technology, tend to distribute in a small range [8].

Given the ability to generate an arbitrary large number of networks with different topologies, an interesting question that arises is whether a specific topology is *structurally* more stable than others. Of course, the topology of existing real-world networks cannot be drastically modified, but to understand which topology is expected to be more robust than others, can be a valuable information for network expansion planning and for wide-area controllers that can be designed to "emulate" a certain topology. This is the research question tackled in this paper.

With this aim, four topologies are considered: random graph; small-world graph; nested small-world graph; and lattice graph. For each topology, we first generate a set of synthetic networks along with steady-state of transmission lines and loads and dynamic data of synchronous machines. Then the parameters of branches and synchronous machines are generated based on adequate distributions. Each network is finally simulated considering random contingencies to test both transient and frequency stability.

II. Generation of Synthetic Networks

To generate synthetic networks, we follows three steps. First, the graph is generated so that it satisfies certain specifications, such as average number of connections and connectivity of each nodes. The nodes of the graph are the buses of the network and the arcs connecting the buses are transmission lines and transformers. Then, a predefined amount of buses are designated as loads and another amount as generators. Finally, the steady-state parameters of the branches, loads and generators, as well as dynamic parameters of synchronous machines are assigned using distributions that resembles real-world networks. The remainder of this section describes these steps.

A. Network Topology

The starting point of the generation of the topology of a synthetic network is a graph $G(n, m)$, where n is the number of nodes and m the number of edges. For the purposes of this work, the graph can be represented trough an $m \times n$ incidence matrix $\mathbf{A}$:

$$\begin{cases} a_{h,i} = 1, a_{h,j} = -1, & \text{if edge } h \text{ connects nodes } i \text{ and } j \\ a_{h,l} = 0. & \text{if} \quad l \neq i, j \end{cases} \tag{1}$$

The topology of a grid network can be fully defined by its $n \times n$ Laplacian matrix, which can be obtained as $\mathbf{L} = \mathbf{A}^T \mathbf{A}$ with

$$\ell_{i,j} = \begin{cases} -1, & \text{if there exists an edge } i - j, \text{ for } i \neq j \\ k, & \text{with } k = -\sum_{i \neq j} \ell_{i,j}, \text{ for } i = j \\ 0, & \text{otherwise.} \end{cases} \tag{2}$$

where k is the number of connections, or *degree* of each node. For synthetically generated graphs, a relevant parameter is the average degree $\langle k \rangle$, that indicates the average number of edges of each node. Finally, let L be the typical distance between any two nodes. Then the average distance l_G can be derived as follows:

$$l_G = \frac{2}{n(n-1)} \sum_{i,j} L_{i,j} \tag{3}$$

The topologies considered in this paper are defined as follows.

1) Random graph: The number of edges, m, is given. Then the existence of every possible edge occurs with probability $0 < p < 1$, with $p = 1/\langle k \rangle$. The theory of random graph has its roots in the pioneering work by Erdős-Rényi [9]. It can be shown that, for a given $\langle k \rangle$, random graphs minimize L.

2) Lattice: Each node is connected only to a constant number of neighboring nodes, i.e., has a $\langle k \rangle = k = \text{const.}$ edges. It can be shown that, for a given k, lattices have the largest L.

3) Small-world graph: Similarly to the lattice, each node has a given number $\langle k \rangle$ of edges to neighboring nodes. A small number of nodes, however, are randomly connected to *remote* nodes. This is the Watts-Strogatz graph model presented in the famous paper [4] which can be shown to have $L = \log(n)$ and, hence, approximate the L as a random graph with same $\langle k \rangle$.

4) Nested small-world graph: This topology is based on the concept of small-world graph. In [6], it has been shown that large power grids, e.g., $n > 300$, do not have exactly the properties of a pure small-world graph. Nodes that are geographically *very far away*, in fact, are very unlikely connected. Reference [6] proposes thus to generate large synthetic power systems as a cluster of r smaller small-world graphs with an average number of nodes $\langle s \rangle$, with $r \cdot \langle s \rangle \approx n$. This kind of networks requires the definition of an additional parameter, namely the average $\langle c \rangle$ connections among clusters.

B. Network Data

Once the topology is defined, one has to populate the grid with the static and dynamic data of branches, loads and generators.

1) Transmission line and transformer parameters: The admittance matrix $\bar{\mathbf{Y}}$ of the electric network is obtained from the incidence matrix $\mathbf{A}$ as:

$$\bar{\mathbf{Y}} = \mathbf{A}^T \mathbf{\Lambda}^{-1}(\bar{z}_L) \mathbf{A} \tag{4}$$

where $\mathbf{\Lambda}(\cdot)$ is a diagonal matrix whose size elements are those of its argument vector, and $\bar{z}_L$ is the vector of the m impedances of the network branches.

The imaginary part of $\bar{z}_L$, i.e., branch reactances, are generated by means of a Gamma distribution [6], whose PDF function is:

$$f(x) = \frac{1}{b^a \Gamma(a)} x^{a-1} e^{x/b} \tag{5}$$

where a and b are the shape and scale factors, respectively. [1] Finally, the resistance of each branch is obtained by multiplying the reactance for a coefficient, say k_r, generated with a uniform distribution in a given range.

2) Steady-state load and generator data: In a typical real-world electric grid, 20-40% are generation buses; 40-60% are load buses and 10-20% are the connection buses [6]. Therefore, in this work, 65% of all the total network buses of the system, n, are loads and 25% of buses are generation which are chosen randomly from total buses in a given topology. The total power generation and consumption are defined as:

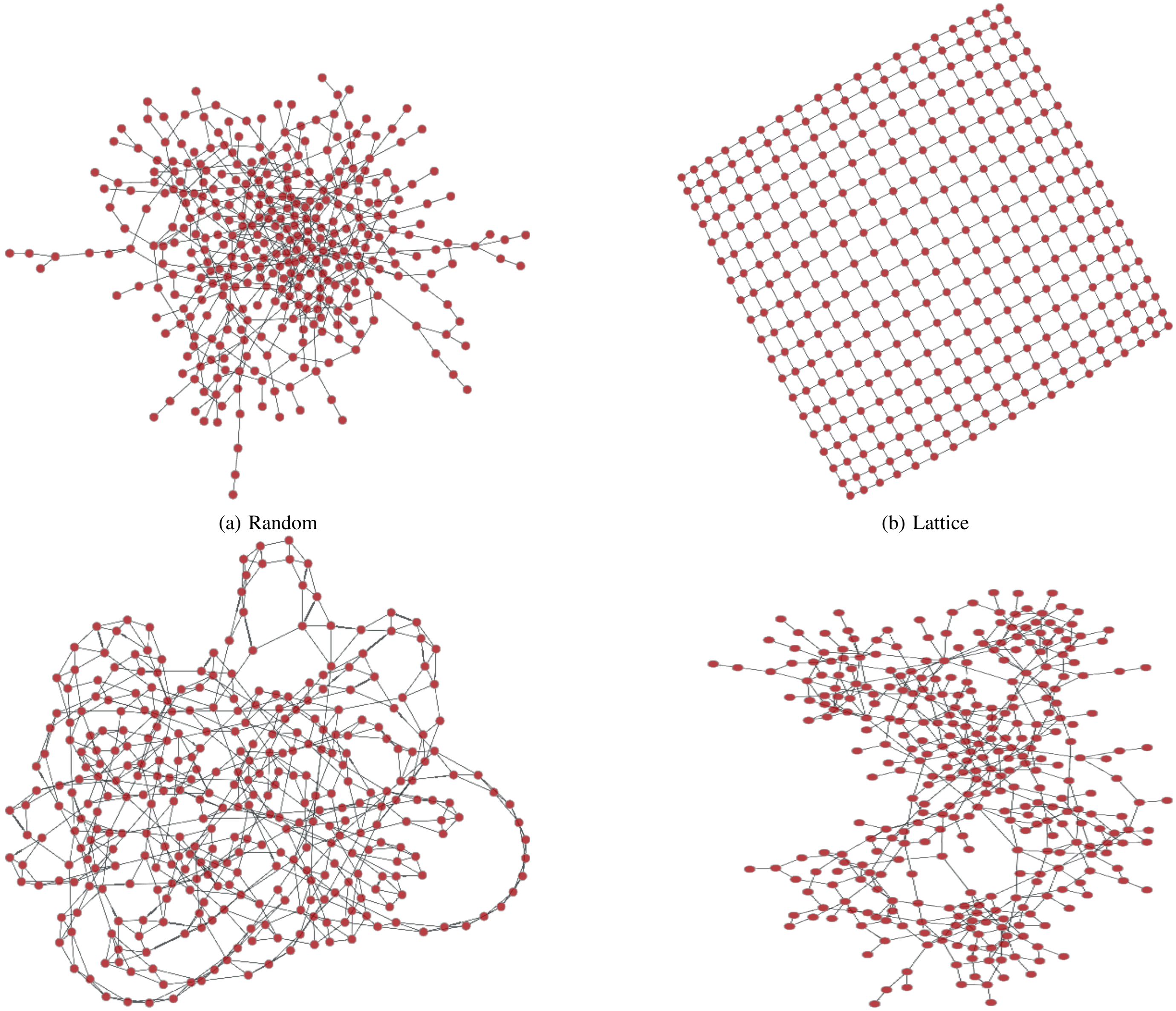
$$p_{G,\text{tot}} = k_G \cdot n, \qquad p_{L,\text{tot}} = (1 - k_L) \cdot p_{G,\text{tot}} \ , \tag{6}$$

where k_G and k_L are heuristic coefficients, chosen so that the probability that the power flow analysis of the resulting synthetic network is feasible is above a certain threshold, e.g., 95%. Then, the power of each generator and load is generated randomly using a given distribution and such that their sums equal $p_{G,\text{tot}}$ and $p_{L,\text{tot}}$, respectively. Finally, load power factors and generator terminal bus voltage magnitudes are generated considering a uniform distribution.

3) Dynamic data: The focus of this paper is on transient and frequency stability analysis. For transient stability analysis, we assume that generators are modeled through the classical second-order model. For frequency stability a fourth order dq-axis synchronous machine model and first order turbine governor and automatic voltage regulators are considered (see [10]).

The rated capacity, inertia H and damping D of synchronous machines are generated based on uniform distributions in a given range deduced considering typical power plant data given in [8]. Other machine and controller parameters are assumed to be the same per-unit average values – again based on [8] – for all machines. Finally, to simulate different

[1] We have also considered other distributions discussed in [6], such as the double-Pareto log-normal distribution, and the shifted geometric distribution, but results are similar to those obtained with (5) and are not discussed here.



(a) Random (b) Lattice

(c) Small-world (d) Nested small-world

Fig. 1: Examples of network topologies considered in the case study ($n \approx 350$). : (a) $G(350, 469)$, (b) $G(361, 684)$, (c) $G(350, 700)$, (d) $G(343, 517)$.

levels of non-synchronous generation, a certain percentage of generators are modeled as distributed energy resources [11].

III. Case Study

The transient and frequency stability responses of the topologies discussed above are tested in this section. The objective of the case study is to determine which topology has a higher expectation to be stable following a random three-phase fault or a random load or generator outage. With this aim, a thousand synthetic networks are generated per each topology and, per each network, a time domain simulation after a random contingency is carried out.

The parameters utilized to generate the synthetic networks are as follows. Topologies are generated assuming $n = 1000$, $\langle k \rangle = \langle c \rangle = 4$, and $r = 5$. Figure 1 depicts representative graphs of the four topologies considered in this work. The shape and scale factors in (5) to generate branch reactances are chosen as $a = 1.88734$ and $b = 0.05856$, respectively. The factor utilized to calculate branch resistances based on the reactances defined above is generated with a uniform distribution in the range $[0.08, 0.12]$. Finally, power flow and relevant synchronous machine parameters and distribution ranges are shown in Table I. For each topology, we consider four scenarios, namely a different percentage of generators, say α, modeled as synchronous machines. The values considered are $\alpha = \{10, 20, 40, 60\}\%$. For each topology and scenario, 500 synthetic networks are generated.

TABLE I: Parameters to generate power flow and dynamic data

Gamma distribution for $\boldsymbol{x}_L$	$a = 1.88734$, $b = 0.05856$
Branch resistance coefficient	$k_r = \text{uniform}(0.08, 0.12)$
Generator active power coefficient	$k_G = 0.142$
Power flow generator voltage	$v_G = \text{uniform}(1.01, 1.05)$ pu
Load active power coefficient	$k_L = 0.0255$
Load power factor	$\cos\phi_L = \text{uniform}(0.9, 1)$
Inertia constants of generators	$H = \text{uniform}(1, 6)$ s
Damping coefficients	$D = \text{uniform}(0.05, 1)$ pu

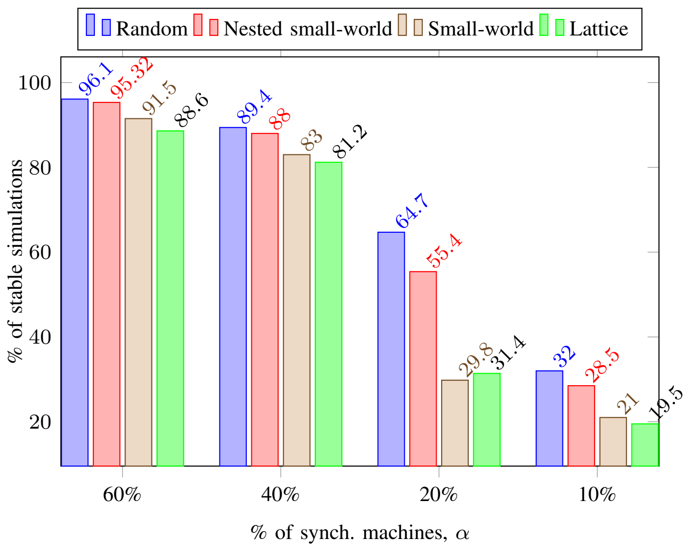


Fig. 2: Transient stability analysis: percentages of stable simulations following a three-phase short-circuit.

All simulations are obtained using Dome, a Python based power system analysis software tool that allows solving large power systems modeled as a set of DAEs [12]. The algorithms to generate the random, small-world, and lattice topologies have been based on existing Python libraries, namely Networkx and Graph-tool. The nested small-world algorithms was implemented *ad hoc* based on [6].

A. Case 1: Transient stability analysis

In this first case, we study the probability of the loss of synchronism for the four considered topologies and different inertia levels, α, following a three-phase fault that occurs at a randomly picked bus and is cleared after 50 ms.

Figure 2 shows the results for the transient stability analysis. It is important to note that results per each topology and inertia level are not indicative *per se*, but only relative to each other. As expected, independently from the topology, the higher the percentage of synchronous machines with respect to the total generation, i.e., the higher the total inertia, the higher the percentage of stable simulations after the fault clearing. The fact is that the random topology is the most stable, while the lattice is the least. Small-world and nested small-world topologies show intermediate properties between the two extremes, being the small-world almost as stable as the lattice.

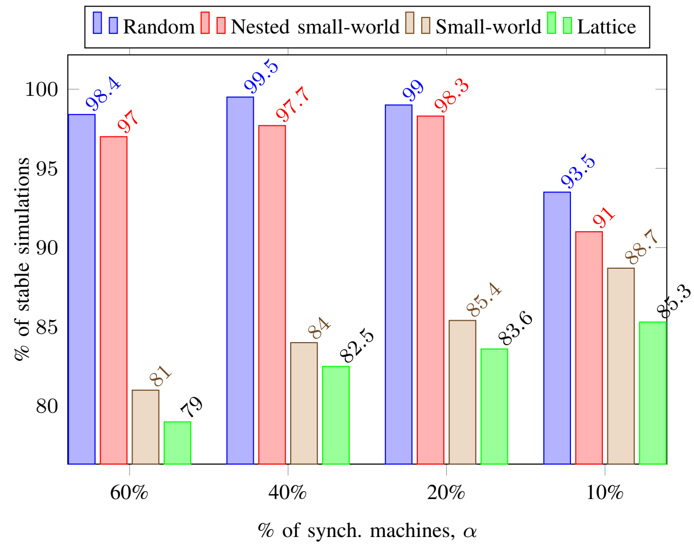


Fig. 3: Frequency stability analysis: percentages of stable, simulations following a loss of load.

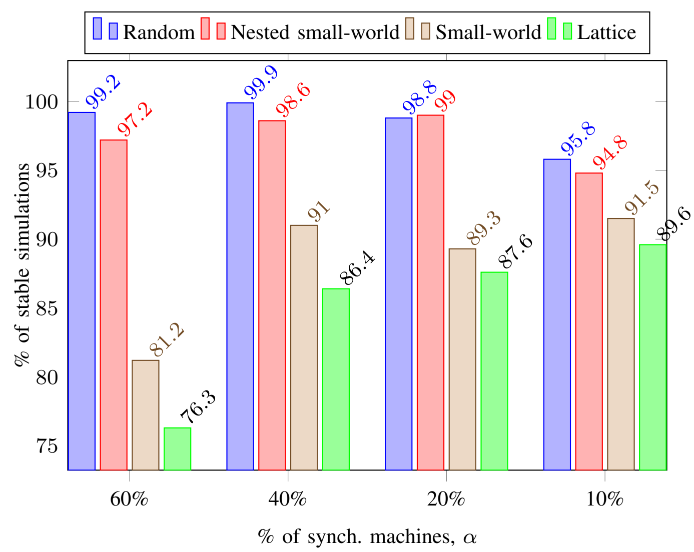


Fig. 4: Frequency stability analysis: percentages of stable, simulations following a loss of generation.

B. Case 2: Frequency stability analysis

In this second case, we consider the frequency response of the four topologies after a loss of load and loss of generation, selected randomly per each network. A simulation is assumed to be stable if 20 s after the contingency, the frequency of the center of inertia of the system is within a given range, namely $[0.984, 1.016]$ pu(Hz).

Figures 3 and 4 show the results for each scenario and inertia level. In this case, the level of inertia is not crucial for the system as the load outage is never causing a loss of synchronism. Results, however, confirm the trend observed for the transient stability analysis, namely, the lower the average distance between nodes, the lower the number of stable simulations.

Conclusions

The case study carried out in this work leads to some remarkable conclusions. From a topological point of view, random graphs are assumed to have more interesting properties than regular lattices. Moreover, from a dynamical perspective, we obtain the same result, namely lattices are less stable that random networks. It is also interesting to observe that small-world topologies, which are typical of medium-size power systems, are similar to random networks in terms of topological properties, and, at the same time, similar to lattices with respect to stability analysis. Hence, the typical structure of medium-size power grids is also almost the *best possible topology*. Large interconnected networks, however, which tends to have a topology similar to a nested-small world, appears to be statistically more stable than smaller ones.

The lower stability of the lattice can be qualitatively explained in terms of its topological property to have the higher average distance between any two nodes [4]. This also means that the dynamic coupling between any two synchronous machines is weaker. On the other hand, random networks have the minimum average distance between any two buses, which leads to a strong dynamic coupling of all machines. Hence, from a dynamic point of view, increasing the interconnections among distant points of a power system can make the system strongr. This conclusion is consistent with respect to the well-known fact that increasing the number of interconnections increases the available transfer capability of an electric network – see for example, recent discussions on the European “super grid”, e.g., [13], a project that is mainly driven by economic considerations.

Future work will focus on the definition of proper control strategies to allow increasing the connectivity of large power systems – which increases the topological randomness of the grid – while retaining the stability of more regular, e.g., small-world, topologies.

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