On the Stochastic Nature of Deterministic Power System Models for Dynamic Analysis

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***Abstract*—The paper presents a study on the impact of uncertainty on the dynamic response of electric power systems. Three sources of uncertainty are considered, namely, (i) uncertainty in the values of the parameters of physical devices; (ii) uncertainty in the models of dynamic devices; and (iii) variations of the parameters and the numerical scheme to integrate the differential algebraic equations that describe the system. A Monte Carlo analysis is used to define the impact of each source of uncertainty as well as all sources together on the dynamic response of the well-known IEEE 14-bus system.**

I. Introduction

In recent years, power systems all around the world have undergone a drastic restructuring that has led, among other aspects, to a significant increase of the capacity of non-dispatchable energy resources, such as wind and photovoltaic power plants. This has brought the attention on the impact of uncertainty on the behaviour of power systems. While uncertainty affects all aspects of the power system at different time scales, the focus of this paper is on transient stability analysis. With this regard, the paper shows a set of simulation results that partially debunk the common belief that deterministic models of power systems are reliable and accurate.

Time domain integration is the backbone of angle and voltage stability analyses of power systems [1]. The high nonlinearity of the power system model, in fact, prevents using the plethora of mathematical tools to predict the response and define the control of linear systems. Moreover, the complexity and variety of devices and controllers that real-world power systems include make impossible in practice to use effectively direct methods based on the Lyapunov second stability criterion. Numerical time-domain integration is thus the only reliable tool to define the dynamic behaviour of power systems in industrial applications. Moreover, the research and implementation of efficient integration schemes [2], [3] and the development of algorithms based on parallel computing [4]–[6] have also drastically reduced the computational burden of such an approach.

The study of the impact of uncertainty on the transient behaviour of power systems is not a particularly active research area. There are, of course, recent studies that consider stochastic processes [7]–[10]. However, there is the common belief, especially by industry, that the transient stability analysis of power systems is relatively robust and reliable. In other words, despite the several simplifications and uncertainties that inevitably affect the model of the power system itself, results are expected to be conservative. Moreover, the level of uncertainty affecting transient stability studies is commonly considered not significant enough to seriously affect the behaviour of the system. In this paper, we show, however, that even neglecting the sources of stochastic processes, the complexity of power system models, i.e., their inherent nonlinearity, along with the uncertainty that affects parameters, models, and numerical methods can actually modify consistently the transient response of power systems.

The analyses carried out in the paper are as follows.

- A Monte Carlo analysis of the effect of parameter and model uncertainty, as well as a sensitivity analysis of the parameters of the numerical scheme to solve the time domain-integration of power systems. This analysis is aimed to identify the impact of each kind of uncertainty alone. Then, the combined effect of all sources of uncertainty together is discussed.
- A discussion on basic statistical properties, e.g., expectation and variance of the trajectories obtained with the Monte Carlo analysis above. Such quantities complement the information obtained with the standard deterministic model and provide a quantitative tool to evaluate the confidence degree of such a model.

The case study presented and discussed in the paper shows that the dynamic response of deterministic power system models has to be considered with caution. While the effect of the uncertainty of each parameter is likely negligible, the combined effect of all uncertainties above can have a drastic impact on the behaviour of the system. This result is certainly a byproduct of the high nonlinearity of power system models. This conclusion and its consequences on the interpretation of power system dynamic simulations is the main contribution of the paper.

II. Power System Model

This section provides a brief overview of the formulation of power system differential-algebraic equations as well as the models of dynamic devices considered in the case study. While the latter are well-known, recalling such models helps clarify the set up of the simulations discussed in Section III.

A. Semi-implicit Formulation

In this paper we use a semi-implicit DAE model with inclusion of discrete events, as follows [11]:

$$\begin{aligned} \boldsymbol{T}\dot{\boldsymbol{x}} &= \boldsymbol{f}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{u},\boldsymbol{\eta}) \\ \boldsymbol{R}\dot{\boldsymbol{x}} &= \boldsymbol{g}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{u},\boldsymbol{\eta}) \end{aligned} \tag{1}$$

where $\boldsymbol{f}$ ($\boldsymbol{f}:\mathbb{R}^{n+m+p+q}\mapsto\mathbb{R}^n$) are the differential equations; $\boldsymbol{g}$ ($\boldsymbol{g}:\mathbb{R}^{n+m+p+q}\mapsto\mathbb{R}^m$) are the algebraic equations; $\boldsymbol{x}$ ($\boldsymbol{x}\in\mathbb{R}^n$) are the state variables; $\boldsymbol{y}$ ($\boldsymbol{y}\in\mathbb{R}^m$) are the algebraic variables; $\boldsymbol{u}$ ($\boldsymbol{u}\in\mathbb{R}^p$) are discrete variables modeling events, e.g., line outages and faults; $\boldsymbol{\eta}$ ($\boldsymbol{\eta}\in\mathbb{R}^q$) are system parameters; and $\boldsymbol{T}$ and $\boldsymbol{R}$ are $n\times n$ and $m\times n$ non-diagonal and non-full rank matrices.

As discussed in [11], the semi-implicit formulation (1) has several formal and numerical advantages with respect to the standard explicit one. The main feature of the semi-implicit formulation that is used in this paper is the ability to seamlessly vary the order of a model by setting to zero the elements of $\boldsymbol{T}$ and/or $\boldsymbol{R}$ in (1). This features consistently simplifies the implementation of Monte Carlo analysis aimed to define the impact of model order approximations discussed in Subsection III-B.

B. Device Models

To simplify the discussion of the case study, machine and regulator models considered in this paper are briefly recalled in this subsection.

1) Synchronous Machine: The synchronous machine model is assumed to be that discussed in [12]. Differential equations in semi-implicit form are as follows:

$$\begin{aligned} \tfrac{1}{\Omega_n}\dot{\delta} &= \omega-\omega_0 \\ M\dot{\omega}+D\dot{\delta} &= \tau_m-\tau_e(\psi_d,\psi_q,i_d,i_q) \\ T'_{d0}\dot{e}'_q+\tilde{T}''_{d0}\dot{\psi}''_d &= -e'_q-(x_d-x'_d)i_d+v_f \\ T'_{q0}\dot{e}'_d-\tilde{T}''_{q0}\dot{\psi}''_q &= -e'_d+(x_q-x'_q)i_q \\ T''_{d0}\dot{\psi}''_d &= -\psi''_d+e'_q-(x'_d-x_\ell)i_d \\ T''_{q0}\dot{\psi}''_q &= -\psi''_q-e'_d-(x'_q-x_\ell)i_q \end{aligned} \tag{2}$$

where parameters are defined as in [12] and

$$\begin{aligned} \tilde{T}''_{d0} &= \frac{(x_d-x'_d)(x'_d-x''_d)}{(x'_d-x_\ell)^2}T''_{d0} \\ \tilde{T}''_{q0} &= \frac{(x_q-x'_q)(x'_q-x''_q)}{(x'_q-x_\ell)^2}T''_{q0} \end{aligned}$$

Note that several models of reduced order can be obtained based on the 6th order model. For example, to obtain a 4th order model, it suffices to impose $T''_{d0}=T''_{q0}=0$. Note also that to reduce the dynamic order of (2), it is sufficient to set to zero a time constant on its left-hand side.

2) Automatic Voltage Regulator: The control scheme considered in this paper is the standard IEEE Type DC1 exciter with transient feedback described in [13], whose semi-implicit formulation is as follows:

$$\begin{aligned} T_m\dot{v}_m &= v-v_m \\ T_b\dot{v}_b+K_a\dot{v}_f &= K_a(v^{\rm ref}-v_m)-v_b \\ T_a\dot{v}_a-T_c\dot{v}_b &= v_b-v_a \\ T_f\dot{v}_f &= K_fv_e-v_f \\ T_e\dot{v}_e &= v_a-(K_e+S_e(v_e))v_e \end{aligned} \tag{3}$$

where v is the terminal bus voltage amplitude of the synchronous machine and the ceiling function S_e is that defined in [14]. The amplifier state variable v_a undergoes an anti-windup limiter, with limits $v_a^{\max}$ and $v_a^{\min}$. All other parameters in (3) are defined in [12].

3) Turbine Governor: The turbine governor scheme considered in this paper is a standard linear model of steam turbines, including a servo, a governor and a reheater. The semi-implicit DAE system that describes this model is as follows:

$$\begin{aligned} T_1\dot{x}_s &= p_{\rm order}+\tfrac{1}{R}(\omega^{\rm ref}-\omega)-x_s \\ T_3\dot{x}_c-T_2\dot{x}_s &= x_s-x_c \\ T_5\dot{x}_r-T_4\dot{x}_c &= x_c-x_r \\ \tau_m &= x_r \end{aligned} \tag{4}$$

where all parameters are defined in [12]. The servo state variable x_s undergoes a windup limiter, with limits $p^{\max}$ and $p^{\min}$.

C. Implicit Time Domain Integration Schemes

In this paper, only implicit time-domain integration schemes are considered as these are known to be numerically more stable and accurate than explicit ones, especially for stiff DAEs [15]–[17]. When using implicit methods, each step of the numerical integration is obtained as the solution of a set of nonlinear equations. Assuming that $\boldsymbol{x}$ and $\boldsymbol{y}$ are known at a generic time t, and given a step length h, the values of $\boldsymbol{x}$ and $\boldsymbol{y}$ at $t+h$ can be obtained by solving the following general expression for implicit integration schemes up to the second order:

$$\begin{aligned} \boldsymbol{0} &= \boldsymbol{T}\cdot\boldsymbol{\xi}-\beta h(\boldsymbol{f}+\kappa\boldsymbol{f}_t) \\ \boldsymbol{0} &= \boldsymbol{R}\cdot\boldsymbol{\xi}-h\boldsymbol{g} \end{aligned} \tag{5}$$

where h is the integration time step, $\boldsymbol{f}_t$ is the known vector of differential equations evaluated at time t, and $\boldsymbol{\xi}$ is a numerical approximation of the time derivative of state variable that depends on the scheme, as follows:

$$\boldsymbol{\xi}=\boldsymbol{x}-\sum_{\ell=1}^{\nu}\gamma_\ell\boldsymbol{x}(t-(\ell-1)h) \tag{6}$$

Table I summarizes the coefficients for the backward Euler method (BEM), implicit trapezoidal method (ITM), and order-2 backward differentiation formula (BDF). These methods are chosen for their different numerical stability properties. BEM and order 2 BDF are L-stable, where BEM can be, in occasions, hyperstable, while ITM is A-stable.

TABLE I
COEFFICIENTS OF THE BEM AND ORDER 2 BDF AND ITM

Scheme	Order	Stability	γ_1	γ_2	β	κ
BEM	1	L-stable	1	-	1	0
ITM	2	A-stable	1	-	0.5	1
BDF	2	L-stable	$4/3$	$-1/3$	$2/3$	0

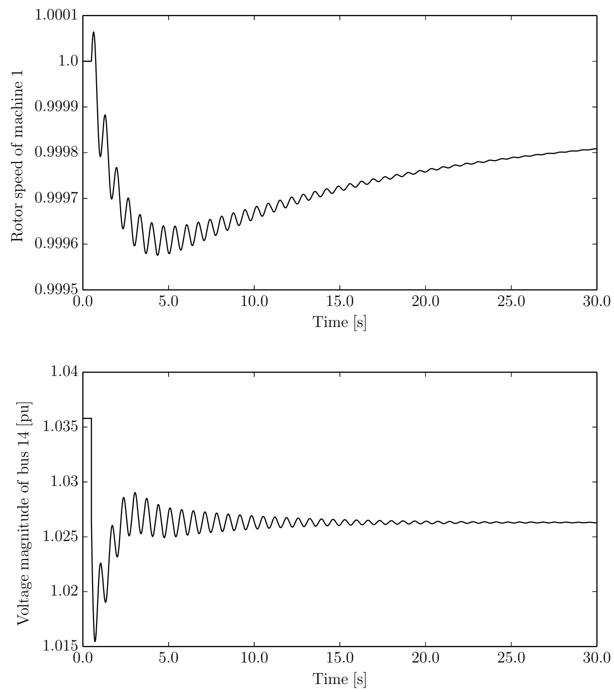

Fig. 1. Trajectories of the rotor speed of machine 1 and of the voltage magnitude at bus 14 for the base-case set of device models and parameters, trapezoidal method and fixed time step $h = 0.025$ s.

III. CASE STUDY

This case study shows, through a Monte Carlo analysis, the impact of parameter, model and numerical integration uncertainty on the transient response of the IEEE 14-bus system. The model of the IEEE 14-bus system considered in this section is that described in [12]. If no power system stabilizer is included, such a system is poorly damped due to the interaction between the subtransient dynamics of the synchronous machine connected at bus 1 and its AVR. The considered contingency is line 2-4 outage occurring at $t = 1$ s. The trajectories for the base-case scenario are shown in Fig. 1. To solve the simulation the ITM with $h = 0.025$ s is used. As shown in Fig. 1, the system is stable and oscillations damp after about 30 s.

In the remainder of this section, the following four scenarios are considered:

- Uniform distribution of device parameter variations.
- Random selection of integration schemes and time steps.
- Random selection of device models.
- The combination of the three scenarios above.

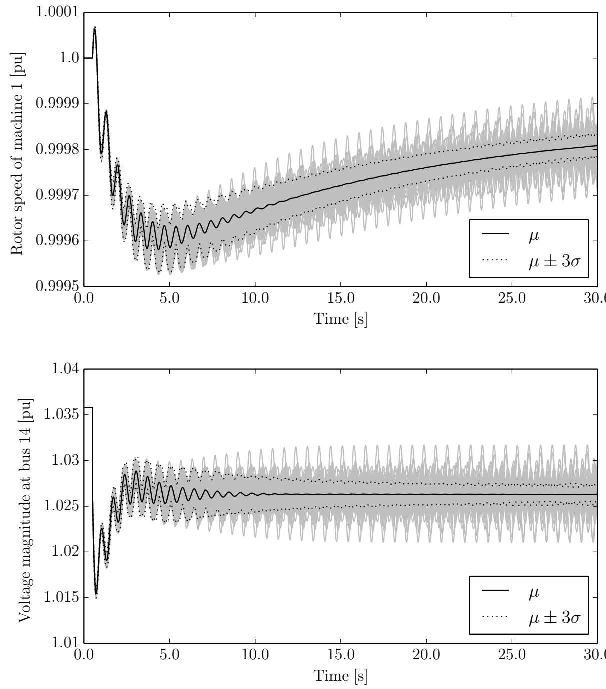

Fig. 2. Trajectories and statistical properties of the the rotor speed of machine 1 and the voltage magnitude at bus 14 considering the effect of device parameter uncertainty.

For each scenario, 2000 time domain simulations have been solved. All simulations are obtained using Dome, a Python-based power system analysis software tool [18] that allows solving time domain analysis in parallel by exploiting multi-core architectures. The Dome version used in this case study is based on Python 3.4.1, Numpy 1.8.2, CVXOPT 1.1.7, ATLAS 3.10.1 and has been executed on a 64-bit Linux Fedora 21 2 Intel 64-bit 6-core 2.66 GHz Xeon X5650 CPUs, and 64 GB of RAM. On such a hardware configuration, Dome completes 2000 simulations in the range of 2 to 5 minutes, depending on simulation settings (e.g., integration time step h).

A. *Effect of Parameter Uncertainty*

For this scenario, all parameters of all static, i.e., transmission lines and loads, and dynamic devices, i.e., synchronous machines and primary regulators, are varied assuming a uniform distribution with boundaries $\pm 2.5\%$ around base case values. Base-case device models and ITM with $h = 0.025$ s are used. Results are shown in Fig. 2. The expected trajectories are similar to those of the base case but slightly better damped, and the standard deviation decreases as the time increases. This is an expected result of probabilistic time domain analysis (see [10] for further details). However, considering the relatively small variation of the parameters, a fairly high number of simulations, namely, about 15%, results in a limit cycle.

B. *Effect of Model Uncertainty*

The effect of uncertainty of dynamic device models on the dynamic response of the system is considered for this scenario.

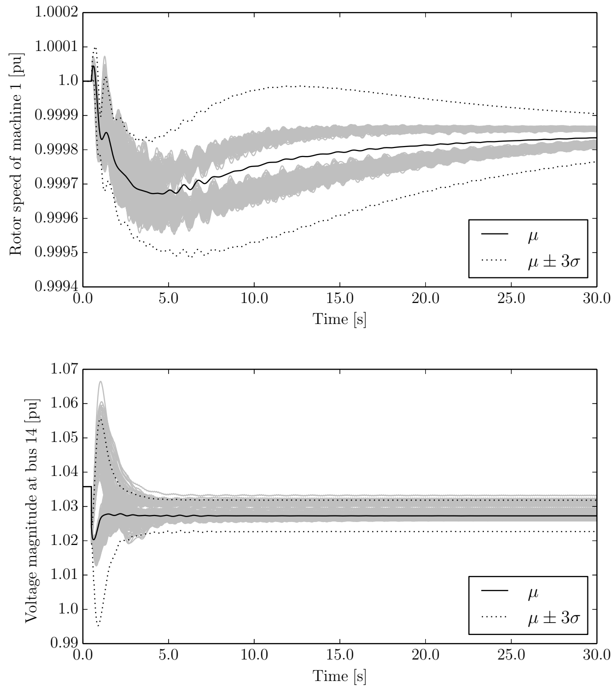

Fig. 3. Trajectories and statistical properties of the trajectories of the rotor speed of machine 1 considering the effect of different device models.

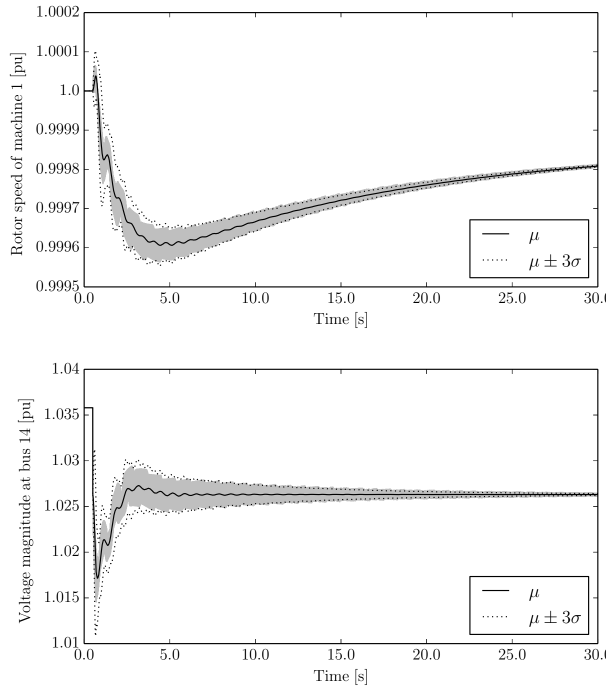

Fig. 4. Trajectories and statistical properties of the rotor speed of machine 1 and the voltage magnitude at bus 14 considering the effect of different different integration methods and settings.

With this aim, the following parameters are randomly assigned either their base case value or zero:

- Model (2): T'_{d0}, T'_{q0}, T''_{d0}, T''_{q0}, $\tilde{T}''_{d0}$, $\tilde{T}''_{q0}$, D.
- Model (3): T_m, T_a, T_b, T_c, T_f, S_e.
- Model (4): T_1, T_2, T_3, T_4, T_5.

Figure 3 shows the results obtained for this scenario. The distribution of the trajectories i such that the standard deviation is not informative. Moreover, rotor angle trajectories appear to grouped into two main clusters. A significant percentage of voltage trajectories, about 50%, deviates from the average one, especially in the first instants following the line outage.

C. Effect of Numerical Uncertainty

For this scenario a random selection of the three time integration methods of Table I as well as of the time step h using a uniform distribution with boundaries $[0.01, 0.3]$ s are considered. Figure 4 shows that the integration scheme and the time step have a small impact on the dynamic response of the system. This result had to be expected due to the robustness of implicit methods.

D. Effect of Parameter, Model and Numerical Uncertainty

In this scenario all uncertainty sources discussed above are considered together. Results are shown in Fig. 5. The main conclusion that can be drawn is that the combined effect of all uncertainty sources is not the sum of each uncertainty alone. The high nonlinearity of the power system model leads to unpredictable trajectories. While the system is apparently always numerically stable, the deviation with respect to the average trajectory can be consistent and lead to unacceptable values of the voltage and/or rotor angles. Hence, the system can be unstable according to the definitions given in [1]. It is also interesting to note that the occurrence of limit cycles is unlikely and that the average trajectory does not oscillates, which can lead to conclude that, on statistical basis, the system is properly damped.

IV. Concluding Remarks and Future Work

The analysis carried out in the paper allows drawing some interesting conclusions on the effect of parameter uncertainties, device models, and numerical simulation. The effect of parameter uncertainty tends to provide uniform and symmetrical distribution. The following are relevant remarks:

- System parameter uncertainty can lead to undamped oscillations although the standard deviation tends to decrease in the long term. Note, however, that undamped oscillations are particularly frequent in the case study as the base case system is poorly damped.
- Device models have a relevant impact on the initial part of the transient and lead to clusters of trajectories.
- The parameters of the numerical integration scheme have a marginal impact on the overall simulation.
- Combining together the effect of all the above, has a dramatic impact on the dynamic response of the system, which is characterized by a high standard deviation during the initial part of the transient, asymmetry of the distribution of the trajectories; and consistent trajectory

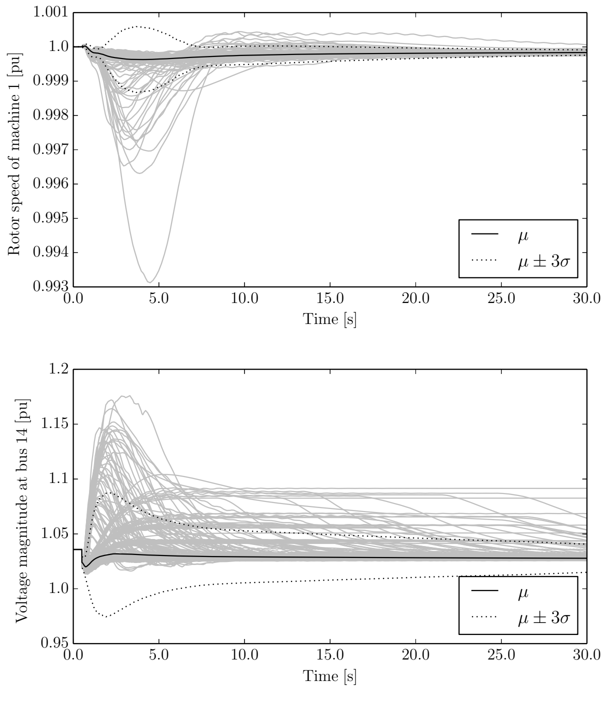

Fig. 5. Trajectories and statistical properties of the rotor speed of machine 1 and the voltage magnitude at bus 14 considering the effect of device parameter uncertainty, different device models and different integration methods and settings.

deviations on a relatively large number of simulations. Even for a light-weight perturbation as that considered in the paper, i.e., a line outage, the uncertainty on what is the expected behavior of the system is surprisingly high.

The observations above suggest to reconsider and, possibly, redefine the usual interpretation of time domain analysis. The computational burden of the analysis presented in this paper makes clearly unfeasible for a practitioner to run similar analyses for each contingency of a real-world system. It is thus necessary to define alternative methods to properly estimate the expected dynamic response of power systems. With this aim, we consider that the following techniques are promising candidates: gray-box modelling approach [19]; Bayesian linear uncertainty analysis [20], and probing techniques for model order selection [21]. Future work will focus on the implementation of efficient and accurate methods to define such estimations for large scale power system models.

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