Small-signal Stability Analysis of Power Systems with Inclusion of Periodic Time-Varying Delays

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***Abstract*—The paper presents a systematic numerical approach to estimate the small-signal stability of power systems with inclusion of periodic time-varying delays. With this aim, the paper first provides the mathematical background of the proposed approach and then discusses the numerical implementation of three types of periodic delays, namely, sinusoidal, sawtooth and square wave. The case study is based on the IEEE 14-bus system with inclusion of a power system stabilizer, whose input signal is assumed to be obtained through a wide-area measurements system. Small-signal stability analysis and time-domain simulation following a large disturbance of the system are presented and discussed.**

***Index Terms*—Delay differential algebraic equations (DDAE), time delay systems, time-varying delays, small-signal stability, wide-area measurement systems (WAMs).**

I. Introduction

A. Motivation

In recent years, the study of the impact of time delays on the stability of power systems has become a relevant research topic. There is a proliferation, in fact, of smart grid applications with a high penetration of communication systems where measured quantities are transmitted to remote control centers. A relevant example of such smart grid solutions are the Wide-area Measurement Systems (WAMs), which are well known to introduce time delays with relatively large magnitudes [1], [2]. Reference [1] indicates that the typical magnitude of communication time delays of WAMs is about 100 ms. According to [2], the maximum magnitude of the time-delays can be up to 700 ms due to communication latency. In this paper, we focus on the small-signal stability analysis of power systems that include WAMs.

B. Literature Review

References [3], [4] have studied the impact of constant time-delays on power system stability through eigenvalue-based approaches and shown that they concern the damping of electromechanical oscillations, lead to severe inter-area power oscillations and can even drive the system to collapse. However, time-delays introduced by WAMs tend to be time-varying [5], which can introduce the Quenching Phenomenon (QP) and have thus a different impact on stability than constant ones.

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QP in delay systems was firstly discussed in [6]. Reference [7] provides the following definition: *QP of a delay system means that if the system is unstable with inclusion of the constant delay* $\tau \in [\tau_{\min}, \tau_{\max}]$*, it may become stable if the constant delay* τ *is replaced by time-varying delay* $\tau'(t) \in [\tau_{\min}, \tau_{\max}]$*, and vice versa.* In other words, assuming that constant and time-varying delays are within the same range, they may have different effects on the system stability.

In some references, time varying delays are treated as stochastic processes [8], [9]. However, according to [5], communication time-delays vary as a periodic (sawtooth) function due to sampling blocks, discrete controllers and zero-order holds that compose the WAMs. The period of these communication delays is around 10 to 20 ms. Since the QP may always occur, modelling the periodic delay as a stochastic one may not lead to accurate results with respect to stability analysis. Hence, a precise time-varying delay model is necessary to fully capture the impact of time-delays introduced by WAMs.

Conventional approaches to investigate the impact of periodic time-delays on the stability of dynamic systems are based on time-domain integration and Lyapunov-Krasovskii Functionals (LKFs). In the field of power system analysis, [10]–[13] are relevant references. The LKF has also been applied for time-varying delays [5], [7], [14]. Frequency-domain approaches have also been investigated [3], [15]–[18]. To the best knowledge of the authors, however, there is no frequency-domain approach to analyze the stability of power systems with inclusion of periodic delays. This paper attempts to fill this gap.

Current frequency-domain approaches require the transformation of the original system into a so-called *comparison system* with equivalent stability characteristics. Reference [19] provides a numerical approach to transform a system with fast time-varying periodic delays into a comparison system with distributed delays. This paper utilizes the technique discussed in [19] to define the comparison system of the DDAEs that describes the power grid.

C. Advantages of the Frequency-Domain Approach

After a careful evaluation of both LKF and eigenvalue-based approaches, we believe that eigenvalue-based techniques are more appropriate to evaluate the small-signal stability analysis of power systems. Relevant remarks are indicated below.

- The LKF theorem provides a sufficient but not necessary stability condition. Thus, the stability assertions obtained through LKF-related approaches tend to be conservative. Although existing fully-fledged LKF approaches [20] can avoid this issue, these are still not a feasible choice for the stability analysis of power systems due to the reasons discussed in the points below. On the other hand, eigenvalue-based approaches provide sufficient and necessary (although local) stability conditions and are thus expected to be more accurate than the LKF. This fact is crucial to tackle the QP.

- The LKF requires the solution of a Linear Matrix Inequality (LMI) problem. The complexity and computational burden of the LMI problem rapidly increase with the system size. This increase is particularly significant for fully-fledged LKF. This drawback of the LKF severely limits its application for real-world realistic-size power system models. Meanwhile, the numerical complexity of obtaining the eigenvalues of larger systems does not increase as much as that of the LMI problem, particularly if only a reduced number of dominant eigenvalues are computed.
- Real-world power networks are complex nonlinear systems and include plenty of uncertain parameters. To make the problem tractable, most LKF approaches applied in power system analysis require to linearize the model at a given operating point [5], [12], [13]. So the main theoretical advantage that is provided by LKF with respect to the eigenvalue analysis, namely, the ability to assert the global stability of a nonlinear systems, is lost.
- Existing studies have already proved that the eigenvalue-based approach can efficiently deal with real-world power systems with inclusion of multiple constant delays [4], [18]. In this paper, periodic delayed equations are transformed into an equivalent set of equations with inclusion of multiple constant delays. Hence, the eigenvalue analysis is expected to be an effective tool to study such systems.

D. Contribution

This paper aims at defining a systematic numerical approach to estimate the eigenvalues of power systems with inclusion of periodic time-varying delays. To this aim, and according to the remarks above, we use a Chebyshev discretization approach [18] to solve a comparison constant multi-delay system that is equivalent to the original equations with periodic delays.

E. Organization

The paper is organized as follows. Section II presents the proposed approach to transform equations with inclusions of fast time-varying periodic delays into equivalent equations with inclusion of multiple constant delays. Section II also outlines how to carry out an approximated small-signal stability analysis of delay systems with inclusion of multiple constant delays. Section III provides a case study based on the IEEE 14-bus system. Finally, Section IV briefly discusses the obtained results, duly draws conclusions, and outlines future work.

II. Proposed Approach

The transformation from a system with inclusion of periodic delays (PDS) into a multiple constant delay system (MCDS) is crucial to enable the small-signal stability analysis carried out in this paper. To this aim, we proceed in the following way. First the PDS is transformed into a system with distributed delays (DDS). Then, the DDS is transformed into an equivalent MCDS. The sequence of transformations is shown in Fig. 1. Subsection II-A describes the first step, which is based on the work proposed in [19]. Subsection II-B describes the second step, namely, how to transform a DDS into a MCDS. Finally, Subsection II-C outlines how to evaluate an approximated solution of the characteristic equation of an MCDS through the Chebychev discretization.

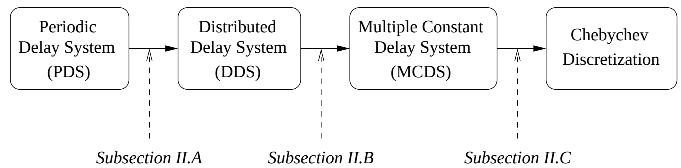


Figure 1: Proposed approach to transform a set of equations with time-varying delays into an equivalent set of equations with multiple constant delays, which is finally linearized and discretized to allow solving the small-signal stability analysis.

A. Transformation of a PDS into a DDS

A key theoretical foundation of the paper is the procedure that allows transforming a linear system that includes *fast* time-varying periodic delays into an equivalent comparison system with inclusion of distributed delays [19]. This subsection aims at briefly summarizing the considered transformation procedure. Detailed mathematical proofs are given in the appendix of [19].

Theorem 1: Consider a linear system with periodic delays:

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}_0\boldsymbol{x}(t) + \sum_{i=1}^{v} \boldsymbol{A}_i\boldsymbol{x}(t-\tau_i(t)) \tag{1}$$
$$\tau_i(t) = \tau_{0,i} + \delta_i f_i(\Omega_i t)\,, \quad i = 1, \ldots, N\,,$$

where $v \in \mathbb{N}^+$; $f_i : \mathbb{R} \to [-1, 1]$ is a bounded periodic function with zero mean and period 2π; δ_i is the amplitude of the periodic delay; and Ω_i is the angular speed of the periodic function. Note that δ_i must be smaller than $\tau_{0,i}$, otherwise negative delays would occur. If all Ω_i are large enough, the small-signal stability of (1) is the same as the following *comparison system*:

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}_0\boldsymbol{x}(t) + \sum_{i=1}^{v} \boldsymbol{A}_i \int_{-1}^{1} w_i(\sigma)\boldsymbol{x}(t-\tau_{0,i}+\delta_i\sigma)d\sigma\,, \tag{2}$$

where $w_i(.)$ is the *weight function* of $f_i(.)$, which can be determined analytically:

$$\int_{-1}^{1} \alpha(t)w(t) = \frac{1}{2\pi}\int_{0}^{2\pi} \alpha(f(t))dt\,, \tag{3}$$

TABLE I: The Weighted Function $w(t)$ and the Characteristic Correction Term $g(s)$ of three types Functions $f(t)$ in (1) [19]. $\partial(\cdot)$ is the Dirac Impulse Functions.

$f(t)$	$w(t)$	$g(s)$
$f_1(t) = \begin{cases} \frac{2}{\pi}(t-\frac{\pi}{2}), & t \in [0,\pi) \\ \frac{2}{\pi}(\frac{3\pi}{2}-t), & t \in [\pi, 2\pi) \end{cases}$ (sawtooth)	$w_1(t) = \frac{1}{2}$	$g_1(s) = \begin{cases} \frac{\sinh(s)}{s}, & s \neq 0 \\ 1, & s = 0 \end{cases}$
$f_2(t) = \sin(t)$	$w_2(t) = \frac{1}{\pi\sqrt{1-t^2}}$	$g_2(s) = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{s\delta \sin(\sigma)} d\sigma$
$f_3(t) = \begin{cases} 1, & t \in [0,\pi) \\ -1, & t \in [\pi, 2\pi) \end{cases}$ (square wave)	$w_3(t) = \frac{\partial(t-1)+\partial(t+1)}{2}$	$g_3(s) = \cosh(s)$

where $\alpha : [-1, 1] \to \mathbb{R}$.

Remark 1: A fast-varying PDS can be transformed into DDS with a known function $f(t)$ that describes its variation.

The characteristic equation of (2) is:

$$\det(s\boldsymbol{I}_n - \boldsymbol{A}_0 - \sum_{i}^{N} \boldsymbol{A}_i e^{-s\tau_{0,i}} g_i(s\delta_i)) = 0 \,, \tag{4}$$

where $\boldsymbol{I}_n$ is the identity matrix with the same size of A_0; $g_i(s)$ is the *correction term* of $f_i(t)$ in the characteristic equation:

$$g_i(s) = \int_{-1}^{1} e^{st} w_i(t) dt \,. \tag{5}$$

Reference [19] provides $w(t)$ and $g(s)$ of three different periodic delays. For clarity, these functions are shown in Table I.

Remark 2: The bound on Ω, which ensures the stability of the PDS (1) equivalent to the DDS (2), can be estimated based on theoretical considerations [21]. Reference [19], however, states that the theoretical bound is conservative and suggests to determine the threshold through numerical tests. Based on a large set of simulations, we have concluded that the threshold is very sensitive to the definition of $f(.)$ as well as to the magnitudes of τ_0 and δ. The eigenvalue analyses carried out in the case study discussed in this paper prove to converge within a range of angular speed values. The correctness of the results is confirmed through time-domain simulations.

B. Transformation of a DDS into an MCDS

This subsection describes a numerical approach to transform a system with distributed delays into an equivalent system with multiple constant delays.

For simplicity, let us consider the following system:

$$\begin{aligned} \dot{\boldsymbol{x}} &= \boldsymbol{F}(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{x}_d, \boldsymbol{u}) \\ \boldsymbol{0} &= \boldsymbol{G}(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{u}) \,, \end{aligned} \tag{6}$$

where $\boldsymbol{F} : \mathbb{R}^{n+m+p} \mapsto \mathbb{R}^n$, $\boldsymbol{G} : \mathbb{R}^{n+m+p} \mapsto \mathbb{R}^m$, $\boldsymbol{x} \in \mathbb{R}^n$ are state variables and $\boldsymbol{y} \in \mathbb{R}^m$ are algebraic variables, and $\boldsymbol{u} \in \mathbb{R}^p$ are discrete variables modelling contingencies, e.g, line outages and faults; and

$$\begin{aligned} \boldsymbol{x_d} &= \boldsymbol{x}(t - \tau(t)) \\ \tau(t) &= \tau_0 + \delta f(\Omega t) \,. \end{aligned} \tag{7}$$

Linearizing the (6) at a stationary solution yields:

$$\Delta\dot{\boldsymbol{x}} = \boldsymbol{A}_0 \Delta\boldsymbol{x} + \boldsymbol{A}_1 \Delta\boldsymbol{x}(t - \tau(t)) \,, \tag{8}$$

where:

$$\begin{aligned} \boldsymbol{A}_0 &= \boldsymbol{F_x} - \boldsymbol{F_y} \boldsymbol{G_y^{-1}} \boldsymbol{G_x} \\ \boldsymbol{A}_1 &= \boldsymbol{F_{x_d}} \,. \end{aligned} \tag{9}$$

From (4), the characteristic equation of (8) is:

$$\det(s\boldsymbol{I}_n - \boldsymbol{A}_0 - \boldsymbol{A}_1 e^{-s\tau_0} g(s\delta)) = 0 \,. \tag{10}$$

Substituting $w_1(t) = \frac{1}{2}$, as shown in Table I, into (2) with $v = 1$, the system with a single fast time-varying sawtooth delay can be transformed as follows:

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}_0 \boldsymbol{x}(t) + \frac{1}{2} \boldsymbol{A}_1 \int_{-1}^{1} \boldsymbol{x}(t - \tau_0 + \delta\sigma) d\sigma \,. \tag{11}$$

Equation (11) describes a DDS, which can be transformed into a MCDS through the Simpson rule. Common Simpson rules include *Alternative Extended Simpson rule*, *Composite Simpson rule* and *Simpson Second rule* [22]. In this paper, we consider the *Simpson Second rule* to approximate distributed delays. However, all the three alternatives above provide similar results.

The formulation of the *Simpson Second rule* is:

$$\begin{aligned} \int_a^b p(x) dx \approx \frac{3h}{8} [&p(x_0) + 3p(x_1) + 3p(x_2) + 2p(x_3) + \\ &\cdots + 3p(x_{N_t-3}) + 3p(x_{N_t-2}) + 2p(x_{N_t-1}) + p(x_{N_t})] \,, \end{aligned} \tag{12}$$

where:

$$\begin{aligned} h &= \frac{b-a}{N_t} \\ x_j &= a + h \,, j = 0, 1, 2, \ldots, N_t \,, \end{aligned} \tag{13}$$

where N_t is the number of sub-intervals of the interval $[a, b]$. As N_t increases, the accuracy of the approximation also increases and finally converges to a constant value. N_t has to be chosen small enough to reduce the computational burden and big enough to ensure convergence and avoid potential rounding errors. Based on our simulations, we found that $N_t = 15$ is a good trade-off between the computational burden and the accuracy.

With the utilization of the *Simpson Second rule*, the implemented comparison system of the (1) with a single fast time-varying sawtooth delay is:

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}_0 \boldsymbol{x}(t) + \frac{3h_1 \boldsymbol{A}_1}{16} \sum_{j=1}^{N_t} \boldsymbol{x}(t - \tau_{1,j}) \tag{14}$$

$$h_1 = \frac{2}{N_t}\,, \tau_{1,j} = \tau_0 - jh\delta\,, j = 0, 1, 2, \ldots, N_t\,.$$

For the periodic delays $f_2(t)$ and $f_3(t)$, directly substituting the weighted functions $w_2(t)$ and $w_3(t)$ into (2) leads to numerical issues. Therefore, a more convenient approach is to consider their corresponding *correction terms* $g_2(s)$ and $g_3(s)$, respectively.

For example, substituting $g_2(s) = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{s\delta \sin(\sigma)} d\sigma$ into (8), we obtain the characteristic equations of the DDS (10) with a single fast time-varying sinusoidal delay:

$$\Delta(s) = s\boldsymbol{I}_n - \boldsymbol{A}_0 - \frac{\boldsymbol{A}_1}{2\pi} \int_{-\pi}^{\pi} e^{-s(\tau_0 - \delta \sin(\sigma))} d\sigma\,. \tag{15}$$

Finally, considering the delay differential equation with characteristic matrix (15), we obtain:

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}_0 \boldsymbol{x}(t) + \frac{\boldsymbol{A}_1}{2\pi} \int_{-\pi}^{\pi} \boldsymbol{x}(t - (\tau_0 - \delta \sin(\sigma))) d\sigma\,, \tag{16}$$

which provides a comparison system with distributed delay. This DDS can be transformed into the MCDS through *Simpson Second rule*. The resulting final comparison system is shown in Table II.

Similarly, substituting $g_3(s) = cosh(s)$ into (10), we obtain the characteristic matrix of (8) with a single fast time-varying square-wave delay:

$$\Delta(s) = s\boldsymbol{I}_n - \boldsymbol{A}_0 - \frac{\boldsymbol{A}_1}{2}(e^{-s(\tau_0 - \delta)} + e^{-s(\tau_0 + \delta)})\,. \tag{17}$$

The implemented comparison system deduced from (17) is also shown in Table II. The characteristic matrix (17) implies a comparison system with inclusion of two constant delays.

We have reproduced the numerical tests of the scalar cases given in [19] using the comparison systems shown in Table II. Our results are consistent with this reference, which confirms the accuracy of the numerical approach proposed in this section.

C. Small-Signal Stability Analysis of an MCDS

Reference [18] discusses a systematic numerical approach to define the small-signal stability of the power system with inclusion of multiple constant delays, which is based on the technique presented in [16]. This technique is required to complete the approach proposed in Section II (see Fig. 1).

Theorem 2: Consider the following set of Delay Differential Algebraic Equations (DDAEs), which describes the transient behaviour of a power system with inclusion of time delays:

$$\begin{aligned} \dot{\boldsymbol{x}} &= \boldsymbol{F}(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{x}_d, \boldsymbol{y}_d, \boldsymbol{u}) \\ \boldsymbol{0} &= \boldsymbol{G}(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{x}_d, \boldsymbol{u})\,, \end{aligned} \tag{18}$$

where the delayed quantities $\boldsymbol{x}_d$ and $\boldsymbol{y}_d$ are:

$$\begin{aligned} \boldsymbol{x}_d &= \boldsymbol{x}(t - \tau_i) \\ \boldsymbol{y}_d &= \boldsymbol{y}(t - \tau_i)\,, \end{aligned} \tag{19}$$

where $i = 1, \ldots, v$ and τ_i are assumed to be constant in the remainder of this section.

The small-signal stability of (18) can be defined through the following linear Delay Differential Equations (DDEs), by linearizing (18) at a stationary solution:

$$\dot{\boldsymbol{x}} = \boldsymbol{A}_0 \boldsymbol{x}(t) + \sum_{i=1}^{v} \boldsymbol{A}_i (t - \tau_i)\,, \tag{20}$$

where matrices $A_i \in \mathbb{R}^{n \times n}$ are obtained based on $\boldsymbol{F_x}$, $\boldsymbol{F_{x_d}}$, $\boldsymbol{G_{x_d}}$, etc., (see [3] and [18] for detail). The characteristic equation of (20) is:

$$\det \Delta(s) = 0\,, \tag{21}$$

where:

$$\Delta(s) = s\boldsymbol{I}_n - \boldsymbol{A}_0 - \sum_{i=1}^{v} \boldsymbol{A}_i e^{-s\tau_i}\,. \tag{22}$$

The characteristic matrix $\Delta(s)$ is transcendental. This means the system (20) has infinite number of eigenvalues. Fortunately, however, to evaluate of the small-signal stability, one only requires a finite number of dominant eigenvalues, e.g., the eigenvalues closer to the imaginary axis. To this aim, one can take advantage of the following *Theorem 3*.

Theorem 3: A finite number of the rightmost eigenvalues of the characteristic matrix $\Delta(s)$ in (21) can be approximated by its corresponding Chebyshev discretization scheme $\boldsymbol{M} \in \mathbb{R}^{(N+1)n \times (N+1)n}$, as shown below:

$$\boldsymbol{M} = \left[\frac{\hat{\boldsymbol{\Psi}} \otimes \boldsymbol{I}_n}{\boldsymbol{B}_N} \right]\,, \tag{23}$$

where $\otimes$ indicates the *tensor product* or *Kronecker product*; $\boldsymbol{I}_n$ is the identity matrix of order n; and $\hat{\boldsymbol{\Psi}}$ is a matrix composed of the first $N-1$ rows of $\boldsymbol{\Psi}$ defined as follows:

$$\boldsymbol{\Psi} = -2\boldsymbol{\Xi}_N / \tau\,, \tag{24}$$

where $\boldsymbol{\Xi}_N$ is the Chebyshev discretization scheme with N nodes.

Matrices $\boldsymbol{B}_N \in \mathbb{R}^{n \times (N+1)n}$ is a set of the linear interpolation of matrices $\boldsymbol{A}_v$ onto the Chebyshev grid, as following:

$$\boldsymbol{B}_N = \begin{bmatrix} \boldsymbol{A}_v & \boldsymbol{0}_{p_{v-1}} & \boldsymbol{A}_{v-1} & \ldots & \boldsymbol{A}_1 & \boldsymbol{0}_{p_0} & \boldsymbol{A}_0 \end{bmatrix}\,, \tag{25}$$

where $\boldsymbol{0}_{p_i}$ is the zero matrix with n rows and p_i columns. The positive integers p_i are allocated according to the Chebyshev grid nodes number N, the number of delays v, and the linear interpolation method.

Remark 3: The Chebyshev discretization scheme is not the only approach to estimate the rightmost eigenvalues of the characteristic equation (21). For example, the Padé approximation [23] and Runge-Kutta methods [15] are alternative

TABLE II: Numerical Approximation Method and Implemented Comparison System of the System with a Single Fast Time-varying Periodic Delay for Small-Signal Stability Analysis. N_t is the number of sub-intervals of the distributed interval, $h_1 = \frac{2}{N_t}$ and $h_2 = \frac{2\pi}{N_t}$, $j = 0, 1, 2 \ldots N_t$; $\tau_{1,j} = \tau_0 - jh\delta$ and $\tau_{2,j} = \tau_0 - \delta \sin(jh)$.

$f(t)$	Approximation of $\boldsymbol{x}(t-\tau(t))$	Implemented Comparison System
$f_1(t) = \begin{cases} \frac{2}{\pi}(t - \frac{\pi}{2}), & t \in [0, \pi) \\ \frac{2}{\pi}(t - \frac{\pi}{2}), & t \in [\pi, 2\pi) \end{cases}$ (sawtooth)	$\frac{1}{2}\int_{-1}^{1} \boldsymbol{x}(t - \tau_0 + \delta\sigma) d\sigma$	$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}_0 \boldsymbol{x}(t) + \frac{3h_1 \boldsymbol{A}_1}{16} \sum_{j=1}^{Nt} \boldsymbol{x}(t - \tau_{1,j})$
$f_2(t) = \sin(t)$	$\frac{1}{2\pi}\int_{-\pi}^{\pi} \boldsymbol{x}(t - \tau_0 + \delta \sin(\sigma)) d\sigma$	$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}_0 \boldsymbol{x}(t) + \frac{3h_2 \boldsymbol{A}_1}{16\pi} \sum_{j=1}^{Nt} \boldsymbol{x}(t - \tau_{2,j})$
$f_3(t) = \begin{cases} 1, & t \in [0, \pi) \\ -1, & t \in [\pi, 2\pi) \end{cases}$ (square wave)	$\frac{1}{2}(\boldsymbol{x}(t - \tau_0 - \delta) + \boldsymbol{x}(t - \tau_0 + \delta))$	$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}_0 \boldsymbol{x}(t) + \frac{\boldsymbol{A}_1}{2}(\boldsymbol{x}(t - \tau_0 - \delta) + \boldsymbol{x}(t - \tau_0 + \delta))$

techniques. In this paper, we choose the Chebyshev discretization because [18] has proven it can achieve the best ratio of accuracy/computational burden through comparing a plenty of approaches. The size of Chebyshev approximation matrix $\boldsymbol{M}$ impacts such a ratio. The estimated rightmost eigenvalues converge to the actual values as N and, hence, the size of $\boldsymbol{M}$ increase. However, [3] and [18] show that N does not need to be high to obtain an accurate estimation.

III. Case Study

In this section, the IEEE 14-bus system model is utilised to illustrate the feasibility and accuracy of the numerical approach discussed above. The topology of the test system is shown in Fig. 2.

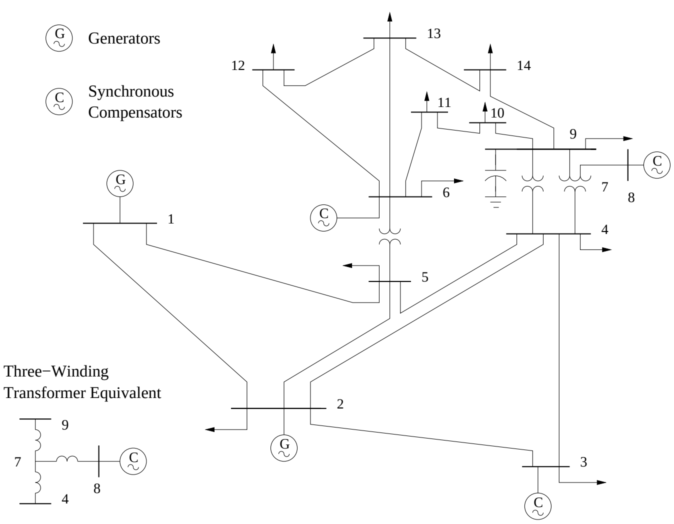


Figure 2: IEEE 14-bus system.

The IEEE 14-bus system includes automatic voltage regulators (AVRs) for each synchronous machine and a remote signal-based power system stabilizer (PSS) connected to the synchronous machine at bus 1. We assume that the remote signal feeding the PSS is sent through a WAM, which leads to non-negligible transmission delay. As a test case, we assume that the WAM communication network is ideal, i.e., the period and amplitude of the time-varying delay is stationary [5]. We considered three cases, namely, the periodical delay varies as a sawtooth $f_1(t)$, a sinusoidal $f_2(t)$ and a square wave $f_3(t)$.

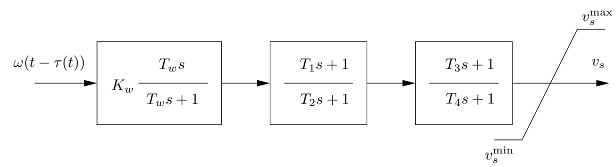


Figure 3: Power system stabilizer control diagram [17].

According to [5], the lower bound of the periodic delays arising in a typical WAM with ideal communication network is zero. Therefore, for all the periodic delays considered in this section, the amplitudes δ_i of the periodic delays are always equal to the constant parts $\tau_{0,i}$, namely:

$$\tau_i(t) = \tau_{0,i} + \tau_{0,i} f_i(\Omega_i t) \, . \tag{26}$$

The control diagram of the PSS is shown in Fig. 3. The input signal to the PSS includes a delayed state variable into the system $x_d = \omega(t - \tau(t))$, where $\tau(t)$ is in the form of (26). The dynamic data of the PSS is shown in Table III. The rest of the system data can be found in [24]. The stabilizer gain K_f and amplifier gain K_a of the AVR of the bus-1 generator are modified as $K_f = 0.00085$ and $K_a = 180$, in order to increase the sensitivity to the delay.

TABLE III: Parameters of the PSS included in the IEEE 14-bus System

Param.	K_w	T_1	T_2	T_3	T_4	T_w	$v_s^{\lim}$
Unit	–	s	s	s	s	s	pu
Value	5.0	0.28	0.02	0.28	0.02	10.0	±0.1

All results in this paper are obtained using DOME [25]. The Dome version utilized in this case study is based on Python 3.6.2 (http://www.python.org), Nvidia Cuda 8.0, Numpy 1.12.1 (http://numpy.scipy.org), CVXOPT 1.1.9 (http://abel.ee.ucla.edu/cvxopt/) and has been executed on a 64-bit Linux Fedora 26 operating system running on a two Intel Xeon 10 Core 2.2 GHz CPUs, 64 GB of RAM, and a 64-bit NVidia Tesla K20X GPU.

The dominating eigenvalues of the IEEE 14-bus system with a remote signal-based PSS are estimated applying the numerical approaches explained in previous Sections. Figure 4 depicts the real part of the rightmost eigenvalues of the IEEE 14-bus system with 20% load increase as a function of the $\tau_0 \in [0, 750]$ ms for different types of fast time-varying

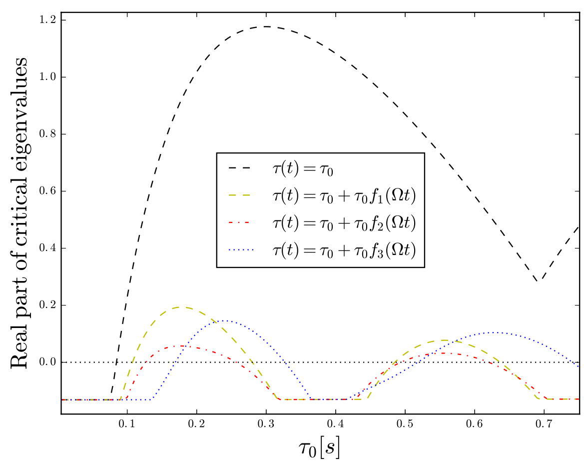


Figure 4: Real-part of the rightmost eigenvalues of the IEEE 14-bus system with 20% load increase as a function of τ_0.

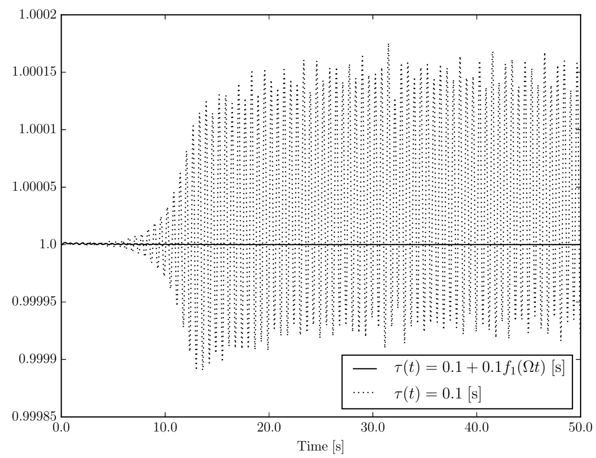


Figure 5: The transient behaviour of bus-1 generator rotor speed of the IEEE 14-bus system with 20% load increase after a small disturbance.

delay. The eigenvalues shown in Fig. 4 are obtained with an integral approximation consisting of $N_t = 15$ pieces (see Section II) and the Chebyshev differentiation matrix $\boldsymbol{M}$ of order 1040×1040 (see Section II-C). All the settings above were found to provide accurate numerical results.

Figure 4 shows that time-delays up to about 80 ms do not have any impact on the stability of the system. Increasing the mean value τ_0, the three time-varying delays generally lead to higher stability margins than the constant delay case, which is a typical effect of the QP. This result also indicates that, if the included delay is time-varying, approximating the delay as constant can lead to conservative results.

The proposed numerical approach studies the system stability through the comparison system (2), which is independent from frequency Ω of the delay itself. Thus, the results of the numerical appraisal can be trusted only if the periodic delay is fast time-varying (see Section II-A). According to the case discussed in [5], we assume that the delay arising in the WAM communication system is varying as a sawtooth wave with period 15 ms, whose corresponding Ω is 418.7 rad/s.

Based on the discussion above, it is worth double-checking the small-signal stability of the actual system (18) with given values of Ω through time domain simulations. With this aim, we consider the following case. For $\tau_0 = 0.1\ s$, the rightmost eigenvalue pair of the system is $-0.0541 \pm j10.8622$, and hence, the system is expected to be stable following a small disturbance. On the other hand, if the power system includes a constant delay $\tau(t) = 0.1$ s, its rightmost eigenvalues are $0.2289 \pm j11.2527$, which indicates that the system is unstable.

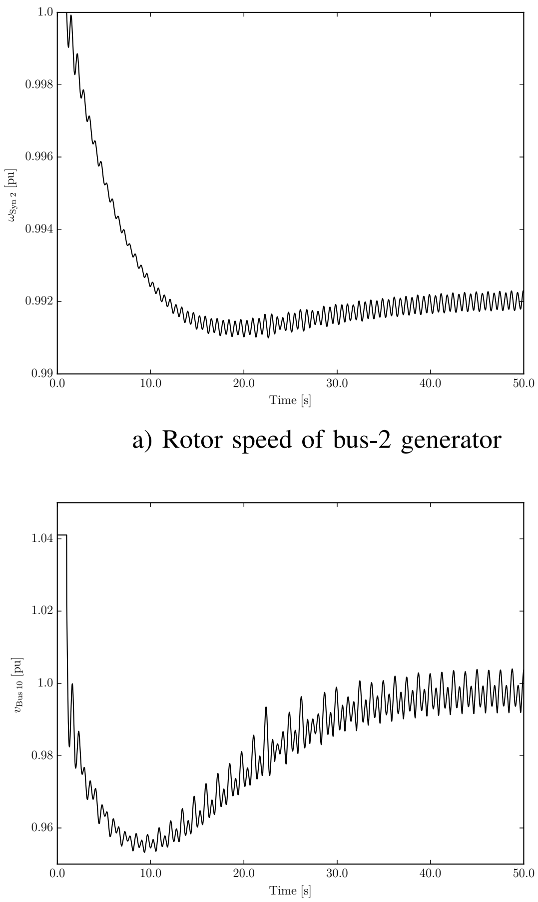
Figure 5 shows the transient behaviour of the IEEE 14-bus system with a 20% load increase following a small disturbance on the rotor speed of the synchronous compensator connected to bus 3. In this case, we consider the constant and sawtooth-varying delays, with $\tau_0 = 0.1$ s and $\Omega = 418.7$ rad/s. The system is stable if the PSS input signal includes the sawtooth delay. On the other hand, the equilibrium point becomes unstable if the delay is constant and the trajectory eventually falls into a limit cycle. Hence, the time domain simulation confirms that the eigenvalues estimated by means of the proposed small-signal stability approach are accurate.

To complete the analysis of the robustness of the proposed small-signal analysis approach that does not take into account Ω, we have carried out the following study. We consider the transient triggered by the outage of line 1-5 and a sawtooth delay $\tau(t) = 0.1 + 0.1 f_1(\Omega t)$ s, and then solve several cases with varying Ω in the range $[314.2, 628.3]$ rad/s, which corresponds to periods ranging from 10 to 20 ms. In all these cases, the estimated post-contingency dominating eigenvalues of the system are $0.1742 \pm j10.5381$. Moreover, the time-varying delays, whose frequency is within the range, the trajectories obtained with time domain integration lead to practically the same transient responses.

Figure 6 shows the trajectories of the rotor speed of bus-2 generator and the voltage at bus 10 following line 1-5 outage. In this case, the trajectories of the system fall into a period-2 limit cycle. This behaviour is consistent with the estimated pre- and post-contingency dominant eigenvalues.

IV. Conclusions

This paper provides a systematic numerical approach to define the small-signal stability of power systems with inclusion of time varying delays. The proposed approach combines two small-signal stability analysis methods: (i) the system is linearized and an equivalent distributed delay set of equations is defined; and (ii) the distributed delays are approximated with a discrete number of constant delays, thus leading to a multiple-delay DDAE. The discretization is obtained by means of the Simpson rule. The stability of the resulting set of equations is finally studied through a Chebyshev discretization scheme.



a) Rotor speed of bus-2 generator

b) Voltage of bus 10

Figure 6: Transient behaviour of bus-2 generator rotor speed of the IEEE 14-bus system with 20% load increase and WAM communication time-delay $\tau(t) = 0.1 + 0.1 f_1(\Omega t)$ s following line 1-5 outage.

The case study based on the IEEE 14-bus system shows the accuracy and numerical robustness of the proposed approach. Moreover, the results of the case study show the existence of quenching phenomenon in power systems. We can thus conclude that accurate delay models are necessary to properly appraise the stability of power systems with inclusion of time-varying delays.

Future work will focus on the application of the proposed technique on real-world large power systems, e.g. all-island Irish grid. Moreover, since the robustness of the proposed approach consistently depends of several approximations, it is relevant to further analysis its numerical accuracy, especially in the case where there are multiple time-varying delays.

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