Validation of the Ornstein-Uhlenbeck Process for Load Modeling Based on μPMU Measurements

Ciaran Roberts
Lawrence Berkeley National Laboratory
cmroberts@lbl.gov

Emma M. Stewart
Lawrence Berkeley National Laboratory
estewart@lbl.gov

Federico Milano
University College Dublin
federico.milano@ucd.ie

***Abstract*—This paper investigates the suitability of the Ornstein-Uhlenbeck process, driven by various Lévy processes, for load modeling at the distribution network level. An in-depth description outlining the procedure for estimating the required parameters is given. Both the statistical properties of the simulated processes and its auto-correlation is compared to that of the field measured data to demonstrate the suitability of the proposed methodology. The development of such stochastic models is facilitated by measures obtained from micro-synchrophasors (μPMU's). The data from these devices serves to demonstrate the need to model the volatility along with validating a model attempting to model said volatility.**

***Index Terms*—Distribution system, Ornstein-Uhlenbeck process, load modelling, micro-synchrophasors (μPMU's), volatility.**

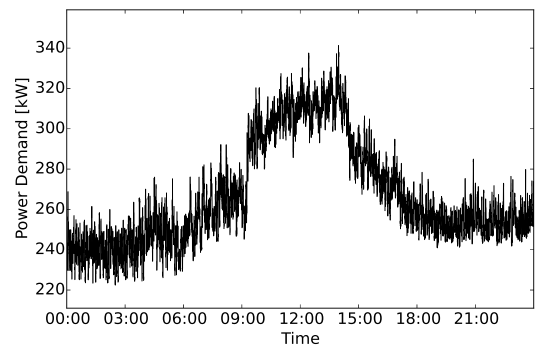

Figure 1. Sample daily load profile from μPMU measurements

I. Introduction

Traditionally, power system analysis has been carried out based on deterministic differential algebraic equations. For this reason, the distribution grid has been modeled through highly simplified equivalent systems or by aggregated voltage dependent loads [1]. This is acceptable, however, only if such aggregate models exhibit limited volatility.

In recent years, the impact of the dynamic behavior of distribution networks has grown considerably due to the emergence of, among others, demand response, intermittent and distributed energy resources, and electric vehicles. This transition is causing a paradigm shift in the role of the distribution grid. While it had once been a passive participant in the delivery of power, it is now set to become an active player.

For these reasons, the power system network is set to move away from a primarily centralized deterministic environment and tend towards a distributed stochastic network. In order to maintain system reliability, power system simulation tools must evolve to capture this stochastic behavior.

The impact of these stochastic sources becomes more evident as the focus shifts from the transmission network to the distribution network. For example, of particular focus for this paper, the level of volatility in the power demand at individual feeders can be quite significant, as shown in Fig. 1. While heuristic techniques are employed to forecast load and minimize the uncertainty, a load modeling practice must be developed to capture the volatility witnessed in Fig. 1.

In this paper, load volatility is modeled based on stochastic calculus and, in particular, the Ornstein-Uhlenbeck process, which is outlined in Section II. For a more thorough treatment of stochastic calculus the interested reader is directed to [2]. Although the Ornstein-Uhlenbeck process has previously been utilized in the literature, a rigorous development and validation of this process has thus far been lacking. This work will serve to demonstrate its suitability along with the derivation of suitable model parameters.

Micro-synchrophasors (μPMU's) are a new measurement device developed by Power Standards Lab (PSL) optimized for distribution and microgrid applications [3]. In this instance, they are utilized for the purpose of recording instantaneous power measurements for the development and validation of the proposed model. These devices are set to address historical issues regarding instantaneous measurements on the distribution network and overcome the technical challenges such as determining the phase angle differences, which on the distribution network are up to two orders of magnitude smaller than those on the transmission network [4]. A brief overview of these devices is given in the Appendix. The devices in question are installed on the campus of Lawrence Berkeley Lab and have a sampling frequency of 120 Hz.

A. Review of Stochastic Load Modeling

Stochastic differential equations (SDE's) have been widely exploited in numerous fields of science and engineering. Their use in power system analysis, however, has thus far been limited. Recent studies have demonstrated the suitability of stochastic calculus for power system analysis, e.g., [5], [6] and [7]. However, the models proposed in these references are not based on real-world measurements and data. One study which successfully demonstrates the suitability of SDE's is [8]. Here, the authors utilize wind speed data from a particular site in order to develop two continuous wind speed models based on the Ornstein-Uhlenbeck process. The resultant models successfully exhibit similar statistical properties to that of the recorded wind data, therefore demonstrating the usefulness of stochastic calculus for power system analysis.

With specific regard to load modeling, there have been previous attempts to utilize stochastic calculus for capturing the volatility of load. One such study is [9] whereby the authors used SDE's, driven by the Wiener process, to add volatility to a deterministic load model attempting to capture the impact of cold-load pick-up on transformer aging. It was concluded that the parameters of the employed Ornstein-Uhlenbeck process had a significant effect on the aging of the transformer. However, the range over which these parameters were varied, coupled with a lack of intuitive understanding regarding the effect of these parameters, limits the usefulness of these results.

The most pertinent load modeling studies in the context of this work are [10] and [11]. In both instances, the authors sought to model electrical load as an Ornstein-Uhlenbeck process, driven by the Wiener process, derived from field measurements. In particular, in [11], the author gives a thorough treatment of load modeling utilizing the Ornstein-Uhlenbeck process. A rigorous development and validation of the process, however, is lacking as the author was restricted by the granularity of the available data. The high sampling frequency of the deployed μPMU's will overcome this difficulty.

B. Contributions

Despite the Ornstein-Uhlenbeck process being proposed and utilized in the past, there exist major limitations in its exploration to date. This work seeks to demonstrate the following:

- The Ornstein-Uhlenbeck process driven by the Wiener process, as has been employed in prior studies [9], is not universally applicable for capturing the volatility of active power demand.
- The Normal-Inverse Gaussian (NIG) process is shown to be more appropriate for capturing the heavy-tails of the distribution of the active power increments.
- The paper also shows that the summation of the Wiener process with a compound Poisson process offers a further improvement again on the NIG process.
- The work is then extended to incorporate the modeling of reactive power through stochastic calculus.

II. Stochastic Calculus

A stochastic differential equation is defined as a process driven by a long term trend, referred to as the *drift*, and stochastic behavior acting upon this drift, referred to as the *diffusion* of the SDE. Such a model is represented as follows:

$$dx = f(x,t)dt + g(x,t)dL(t) \tag{1}$$

where $f(x,t)$ is the drift function and $g(x,t)$ is the diffusion of the Lévy process, $L(t)$. In the multi-dimensional case, $g(x,t)$ is a co-variance matrix which dictates how the vector of Lévy processes enter the system.

Lévy processes are continuous-time stochastic processes with independent stationary increments. The Wiener process and the Poisson process are examples of Lévy processes. A real-valued stochastic process $L(t), t \in [0, \infty)$, is a Lévy process if the following is satisfied [12]:

1) X_o = 0
2) *Independent increments*: For every increasing times $t_o ... t_n$, the random variables $X_{t_o}, X_{t_1} - X_{t_o}, ... X_{t_n} - X_{t_{n-1}}$ are independent;
3) *Stationary increments*: The distribution of X_{t+h} — X_t does not depend on t;
4) *Stochastic continuity*: $\forall \epsilon > 0$,$\lim_{h\to 0} P(|X_{t+h} - X_t| \geq \epsilon)$= 0

The SDE in (1) can be formally integrated to obtain:

$$x(t) = x(t_o) + \int_{t_o}^{t} f(x(t),t)dt + \int_{t_o}^{t} g(x(t),(t))dL(t) \tag{2}$$

The first integral on the right hand side of (2) can be determined with classical calculus such as the Riemann-Stieltjes integral however the same cannot be said about the second integral. Due to the unbounded variation associated with stochastic processes, the Riemann-Stieltjes integral fails to converge. An alternative approach is therefore necessary. In this instance, the Itô integral is the chosen solution.

SDE's of the form presented in (1) exhibit unbounded variation. In the context of load modeling, unbounded variation is not an accurate representation of the loads behavior. Although power demand does exhibit stochastic behavior, this behavior tends to manifest itself as volatility in the vicinity of the forecasted demand. The stochastic process most reflective of this is the Ornstein-Uhlenbeck process, as follows:

$$dx_t = \gamma(\mu - x_t)dt + dL(t), \qquad \gamma \geq 0 \tag{3}$$

where γ is the *mean reversion rate*, which forces the stochastic behavior of the load to remain somewhat close to the long-term mean, μ. Such a process is typically driven by the Wiener process, however, for the purpose of this study the process will be extended to be driven by more generalized Lévy processes.

The solution of (3) for the case of the Wiener process is:

$$X_t = \mu + (x_o - \mu)e^{-\gamma t} + \sigma \int_0^t e^{-\gamma(t-s)} dw_s \tag{4}$$

whereas the expected value E and variance Var are given by:

$$E[X_t | X_o = x_o] = \mu + (x_o - \mu)e^{-\gamma t} \tag{5}$$

$$\mathrm{Var}[X_t | X_o = x_o] = \frac{\sigma^2}{2\gamma}(1 - e^{-2\gamma t}). \tag{6}$$

III. Proposed Method

For the purpose of this study, the Ornstein-Uhlenbeck process is utilized solely for modeling the volatility of the load. Therefore to accurately validate its capabilities to do so, the uncertainty in the forecasted demand must be minimized. In order to achieve this the forecasted demand will be taken as the mean of measured daily profiles at 15 minutes intervals.

In order to correctly characterize the background driving process, we determine first the distribution that best fits the increments of the active power. This process has been theorized to be the Wiener process [8], [10], [11]. In the context of power system modeling the Wiener process may be a suitable driving noise process at higher loading levels, such as at the transmission level, which was of interest in [8], [10], [11].

At the distribution level, however, individual end-uses can carry a greater weighting in the the behavior of the power increments. A normal distribution may not be able to accurately capture these increments. Measured distributions frequently exhibit heavy tails (see, for example, Fig. 2), which cannot be replicated by the Wiener process [13], [14]. In such instances, alternative processes must be considered.

This work both analyzes the suitability of the Wiener process and investigate alternatives which improve the model performance. As the volatility of the load is expected to vary throughout the day, dependent on occupancy, the day is segmented into hours. Appropriate parameters for each of the processes of interest are estimated for each of these hours.

There are a number of methods for estimating the mean reversion rate, γ. However, many of these suffer from bias. In order to improve accuracy, a martingale estimating function proposed in [15] and implemented in [16] for modeling temperature is utilized in this paper.

Based on the method above, an estimation of the value γ is given by a zero of the equation:

$$G_n(\gamma) = \sum_{i=1}^{n} \frac{\dot{b}(X_{i-1};\gamma)}{\sigma_{i-1}^2}\{X_i - E[X_i|X_{i-1}]\} \tag{7}$$

whereby

$$b(X_t;\gamma) = \frac{d\mu_t}{dt} + \gamma(\mu_t - X_t) \tag{8}$$

The expected value of the Ornstein-Uhlenbeck process is given by

$$E[X_t|X_{t-1}] = \mu_t + (x_{t-1} - \mu_{t-1})e^{-\gamma} \tag{9}$$

Therefore

$$G_n(\gamma) = \sum_{i=1}^{n} \frac{\mu_{i-1} - X_{i-1}}{\sigma_{i-1}^2}\{X_i - (X_{i-1} - \mu_{i-1})e^{-\gamma} - \mu_i\} \tag{10}$$

for which the unique zero of (7):

$$\gamma = -log\left(\frac{\sum_{i=1}^{n} Y_{i-1}\{X_i - \mu_i\}}{\sum_{i=1}^{n} Y_{i-1}\{X_{i-1} - \mu_{i-1}\}}\right) \tag{11}$$

where

$$Y_{i-1} = \frac{\mu_{i-1} - X_{i-1}}{\sigma_{i-1}^2} \tag{12}$$

In the case study discussed in the following section, to validate the proposed methodology, the statistical properties of the realizations generated via the Ornstein-Uhlenbeck and its auto-correlation is compared with that of the measures from the μPMU's.

The simulation of SDE's is a subject which has been thoroughly investigated in prior literature. The interested reader is directed to [17] and [18] for a thorough exploration of the simulation of SDE's. For the purpose of this work the sample realizations were generated using Euler-Maruyamascheme, with a total of twenty days being generated, corresponding to 1,728,000 individual values for each of the investigated processes.

IV. Case Study

The sampling rate of the installed μPMU's is 120 Hz. Each individual sample is comprised of the instantaneous voltage and current phasors for each individual phase. The active and reactive power was calculated and served as the inputs to the proposed model. For the purpose of this study the data is down-sampled to one second intervals. A total of 20 weekdays worth of data was collected.

As discussed in the previous section, the appropriate background-driving Lévy process is determined first. Then, sample realizations of the Ornstein-Uhlenbeck process are generated and the statistical properties of these realizations are compared with that of the measured data in order to validate the proposed model.

Subsection IV-A below rigorously explores the active power. An application and necessary adjustments of the proposed methodology for the reactive power and analysis of results is discussed in Subsection IV-B.

A. Active Power

1) Characterizing the Driving Process: Fig. 2 shows a distribution of the active power increments for a sample hour. The corresponding normal distribution is overlaid. The data appears to be normally distributed. The corresponding normal distribution, however, reveals the distribution to be a fat-tailed distribution. The normal distribution poorly fits the data in this instance and questions the validity of the Wiener process as the background driving noise in this instance.

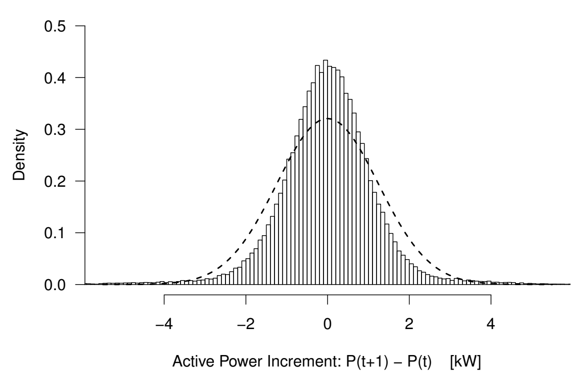

Figure 2. Active power increments with corresponding normal distribution

A more representative distribution for the increments of the power is the Normal-Inverse Gaussian (NIG) distribution. Such a process has been widely investigated for capturing the heavy tails of observed data [14] [19]. If N has a distribution described by the NIG distribution, its typically noted as:

$$N \sim \text{NIG}(\alpha, \beta, \delta, \mu) \tag{13}$$

where α is a steepness parameter, β is an asymmetry parameter, δ is a scale and μ is the location parameter. The variance of the NIG process is given by:

$$\sigma^2 = \frac{\delta\alpha^2}{(\alpha^2 - \beta^2)^{3/2}} \tag{14}$$

For the purpose of this work, $\beta = 0$ and $\mu = 0$ are assumed. These values lead to a symmetrical distribution centered on 0.

The Ornstein-Uhlenbeck process takes the form of (15) when driven by the NIG process, $n(t)$. The NIG process is simulated by randomly sampling from a NIG distribution with specified parameters, α and δ.

$$dx_t = \gamma(\mu - x_t)dt + dn(t) \tag{15}$$

Fig. 3 shows the data with both a normal and NIG distribution overlaid. The NIG distribution was fit to the data utilizing the R package `ghyp` [20]. The NIG distribution is a significant improvement on the normal distribution.

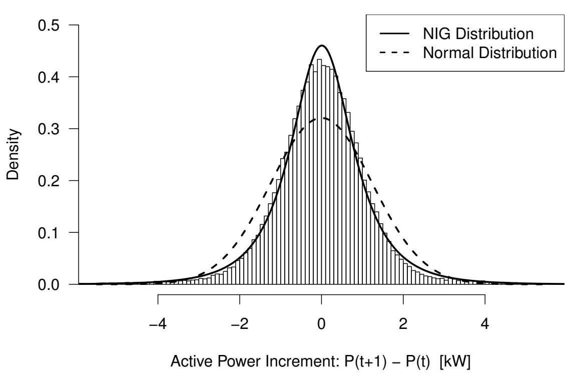

Figure 3. NIG distribution fit to active power increments

In order to asses the capability of both these distributions on capturing the heavy tails of these distributions the log of the density is examined. This is shown in Fig. 4. The NIG distribution, as expected, is an improvement in fitting these fat tails however it is not able to fully capture said tails. Figure 4 demonstrates the unsuitability of the Wiener process for modeling the power demand of this particular feeder. Hence, while the Wiener process may well be suitable for specific feeders, the assumption that it is suitable for the majority of feeders is unsubstantiated.

The importance of accurately modeling the extreme values of these fat-tailed distributions is set to increase as the level of stochastic behavior increases on the distribution network. As DG, particularly in the form of PV, increases on the distribution network it becomes increasingly critical to understand how the stochastic behavior of these resources propagates through the network. Such an understanding would allow for the identification of the optimal localized control strategy. This is particularly pertinent for microgrids which may have islanded themselves from the grid.

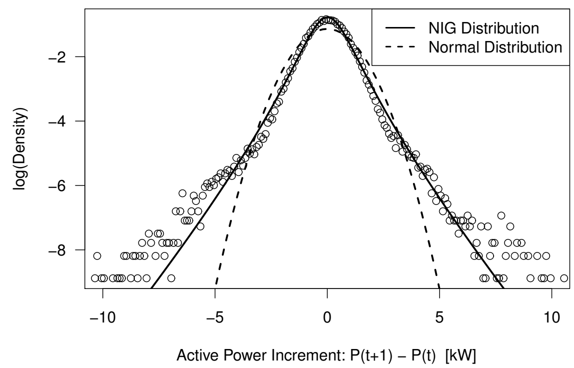

Figure 4. Evaluation of the ability to capture tails of measurement data through the Gaussian and NIG distributions.

To accurately model the heavy-tails shown in Fig. 4 is important to properly capture the volatility of the data. A further improvement on the NIG process is thus desirable. This improvement comes from considering the driving process as a jump-diffusion model. That is, considering the process as a sum of the Wiener process and a compound Poisson process. These types of models are widely considered in stock prices which are susceptible to sudden price spikes. Such a model has also been employed for modeling electricity prices [21], [22]. A jump-diffusion model has some appealing intuitive aspects for modeling the driving noise at lower loading levels. The Wiener process captures the aggregation of minor load variations, e.g., switching of lighting, appliances increasing their power consumption, while the Poisson process captures the discrete switching of large power consumers, e.g., air conditioning or motors.

The resultant proposed Ornstein-Uhlenbeck model takes the form:

$$dx_t = \gamma(\mu - x_t)dt + \sigma dw(t) + J_t dq_t \tag{16}$$

where J_t is a random variable given by an appropriate distribution describing the jumps and q_t is a Poisson random variable with intensity λ such that

$$dq_t = \begin{cases} 1 & \text{with probability } \lambda dt \\ 0 & \text{with probability } 1 - \lambda dt \end{cases} \tag{17}$$

In this paper the jumps are modeled by means of a generalized Pareto distribution as this best fits experimental μPMU data.

In order to calibrate the jump diffusion model, the driving process must be decomposed into an appropriate Wiener and compound Poisson process. This is achieved by utilizing the parameters of a t-distribution as a proxy for a more appropriate normal distribution tuned to the peak. Following this, any increment which lies outside 3 times the standard deviation is classified as a jump. The λ parameter is given as the rate of these jumps while a distribution is fitted to the jumps themselves.

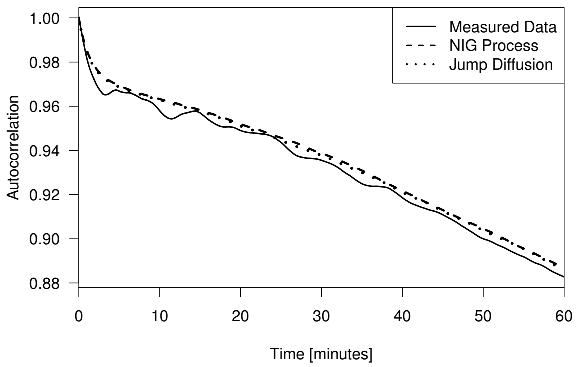

Figure 5. Comparison of the active power auto-correlation of experimental and simulated processes

2) Analyzing Model Performance: The field-measured data was utilized to calibrate both processes of interest. A mean reversion rate of 0.0125 was estimated. A larger value for the mean reversion rate corresponds to more sharp oscillations around the mean while a lesser value tends to facilitate a more gradual return towards the mean. The mean-reversion rate is comparable to the decay constant in the behavior of the half-life phenomenon associated with physics whereby the mean can be equated with being a stable state.

The auto-correlation of the generated realizations is compared with that of the recorded data. The auto-correlation corresponding to lags up to one hour was calculated. At lags beyond this, the underlying trend, rather than the volatility, was deemed to be primarily responsible for the auto-correlation. Fig. 5 shows the auto-correlation of both processes. Both process closely model that of the recorded data with the jump-diffusion model marginally out-performing the NIG process. This was expected as the NIG process under-performed in replicating the larger increments. The auto-correlation, however, only partly validates the processes.

Fig. 6 shows the cumulative distribution function of the incremental active power generated by relations of the processes. Both the NIG process and the jump-diffusion closely mirror the smaller measured increments within the range investigated in Fig. 6.

The motivation for considering both the NIG and jump-diffusion stemmed from the existence of heavy-tails. Therefore to benchmark their respective performance in capturing these tails Fig. 7 focuses on the larger absolute active power increments. The NIG process approaches the field measurements, although as the log-scale histogram suggested in Fig. 3, it only partly succeeds in re-creating the larger jumps. The jump-diffusion model, in this instance, is the process most successful in recreating the larger measured increments.

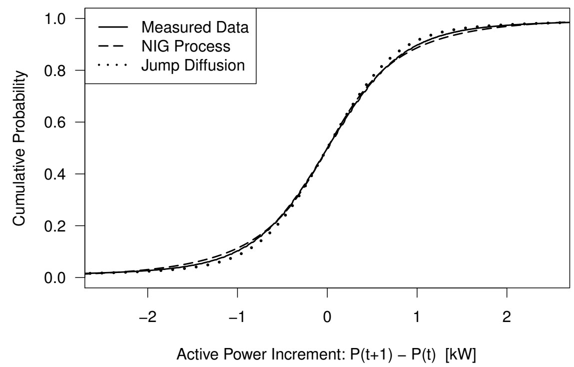

Figure 6. Sample plot of the density percentage active power difference

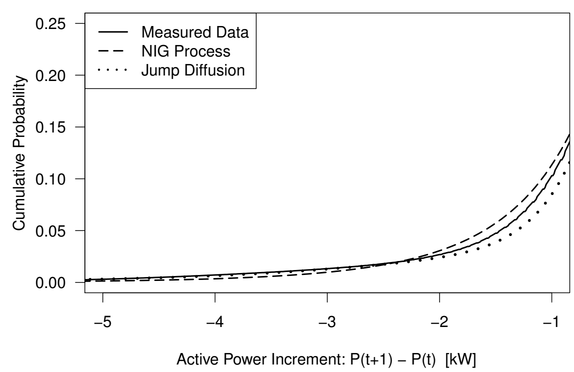

Figure 7. Sample plot of the density percentage active power difference

Fig. 8 compares the cdf of the normalized active power of each of the processes with that of the measured active power. In each case, the profiles were normalized about the fifteen minute mean of the measured active power. Both the NIG and dump-diffusion process show a goo match with the measured data, with the jump-diffusion a slight improvement on the NIG process.

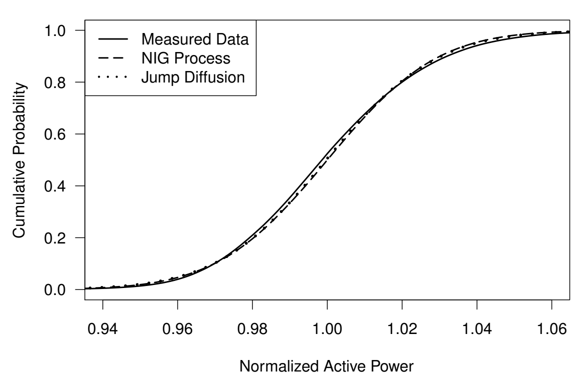

Figure 8. Comparison of experimental and simulated cumulative distributions of normalized active power

Fig. 9 shows a sample realization, for a one hour period, of the Ornstein-Uhlenbeck process driven by the NIG process. Both the forecasted demand along with the output of the Ornstein-Uhlenbeck process are presented in order to develop an intuitive understanding of the process.

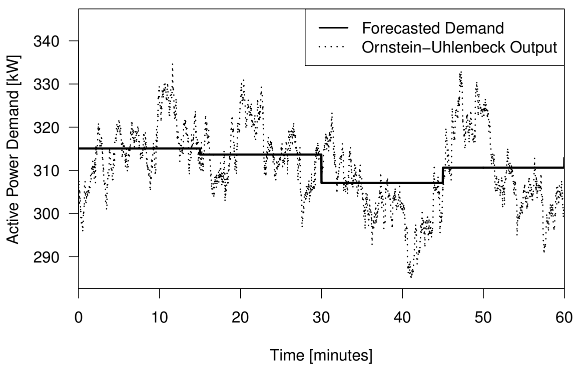

Figure 9. Comparison of experimental and simulated auto-correlations of reactive power

B. *Reactive Power*

The proposed methodology is extended in this section in an effort to model the reactive power. The reactive power exhibits volatility dissimilar to that of the active power and thus is treated separately. Figs. 10 and 11 compare the cumulative distribution function of the simulated data to that of the measured data for both the simulated values and associated increments respectively. Both figures demonstrate the suitability of the Ornstein-Uhlenbeck process driven by both the NIG process and the jump-diffusion process for modeling reactive power.

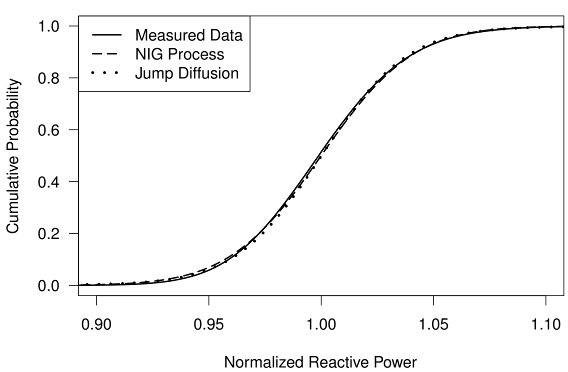

Figure 10. Comparison of experimental and simulated cumulative distributions of normalized reactive power

A comparison of the auto-correlations, however, reveals the NIG process to be the more suitable process in the context of modeling reactive power. The process accurately captures the autocorrelation of the field measured data. This may stem from the reactive power being less susceptible to larger jumps and that the deployment of a jump- diffusion model may

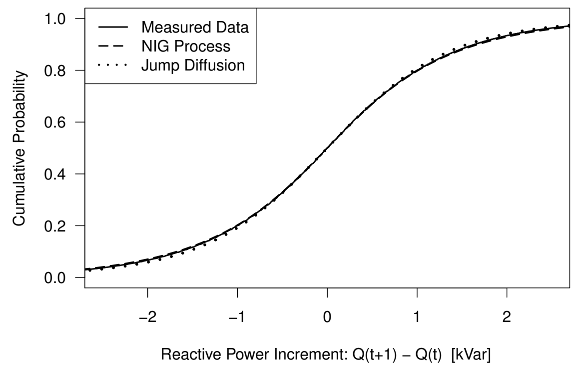

Figure 11. Comparison of experimental and simulated cumulative distributions of normalized of the increments of reactive power

result in an ill-fitting distribution attempting to describe the larger jumps. Also it was noted that there larger jumps usually manifest themselves as large positive increments followed almost immediately by a negative increment of similar size, similar to the behavior of inrush current. Therefore there is a level of deterministic behavior in these jumps that is not captured by the jump-diffusion model. The NIG process does not suffer this fate as it fails to capture these outlying jumps due to their infrequency.

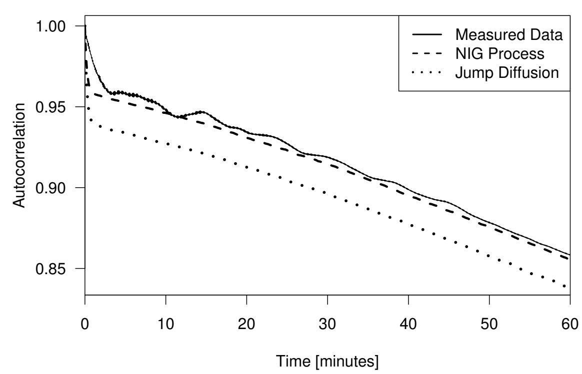

Figure 12. Comparison of experimental and simulated auto-correlations of reactive power

A potential advancement on the modeling of the reactive power utilizing the NIG process would incorporate the effect of inrush current from motor starts. Similarly, other events, could be re-created utilizing stochastic calculus at their respective frequencies once they are recognized in measured data.

V. Conclusions

This paper explores the suitability of stochastic calculus for capturing the volatility witnessed at lower loading levels. As interest in distribution system analysis continues to grow with the deployment of distributed energy resources, the ability to replicate the load behavior increases in importance.

The comparison of experimental data and simulation results shows that the Ornstein-Uhlenbeck process accurately models the behavior of electrical load at a feeder level. The case study also shows that the Wiener process may not be a suitable driving process, as had been previously been proposed in prior studies. Instead the NIG process may be a more suitable driving noise process in an attempt to capture the heavy-tailed distributions of the active power increments. A jump-diffusion model, a summation of a Wiener process and compound Poisson process, offers a further improvement in the context of active power.

The extension of the proposed similarly reveals the capabilities of the proposed method for modeling the reactive power. In this instance, however, the NIG process revealed itself to be a more suitable driving noise process.

Potential future work following on from this study include the investigation of various feeders with different customer mixes and determining whether the NIG process is universally a suitable driving process or whether the Wiener process may suffice for certain customer mixes.

The use of stochastic calculus can also be extended to recreating events in order to ensure system security, Once events have been identified from measurement data, they can be recreated with mirroring probabilities in power system studies utilizing stochastic calculus which would facilitate a more accurate time evolution of the trajectory of the system.

It's critical that stochastic calculus and current modeling techniques not be thought of mutually exclusive approaches, but rather complimentary. A load model, or similar, could be the sum of numerous periodic waveforms, e.g. representing the cycling of air conditioners, and a stochastic driving noise process, representing the human interaction with buildings.

ACKNOWLEDGEMENTS

This work was conducted in the Electricity Research Centre, University College Dublin, Ireland, which is supported by the Electricity Research Centre's Industry Affiliates Program (http://erc.ucd.ie/industry/). This material is based upon works supported by the Science Foundation Ireland, by funding Federico Milano, under Grant No. SFI/09/SRC/E1780. The opinions, findings and conclusions or recommendations expressed in this material are those of the author and do not necessarily reflect the views of the Science Foundation Ireland. The first and third authors has also benefit from the financial support of EC Marie Skłodowska-Curie Career Integration Grant No. PCIG14-GA-2013-630811.

This research is sponsored in part by the U.S. Department of Energy ARPA-E program (DE-AR0000340).

APPENDIX
μPMU'S

The μPMUs used in the LBNL test network were developed by Power Standards Laboratory [3]. μPMUs can capture portions of the operating state of the system to provide actionable intelligence in real-time. Each individual sample encompasses the magnitude and phase angle of the voltage and current phasors for all three phases individually and the GPS time stamp.

REFERENCES

[1] V. Knyazkin, C. A. Cañizares, and L. H. Söder, "On the Parameter Estimation and Modeling of Aggregate Power System Loads," *IEEE Transactions on Power Systems*, vol. 19, no. 2, pp. 1023–1031, May 2004.

[2] B. Øksendal, "Stochastic Differential Equations," in *Stochastic Differential Equations*, ser. Universitext. Springer Berlin Heidelberg, 2003, pp. 65–84.

[3] "Power Standards Lab - PQube Phasor Measurement Unit." [Online]. Available: http://pqubepmu.com/

[4] A. von Meier, D. Culler, A. McEachern, and R. Arghandeh, "Micro-synchrophasors for distribution systems," in *IEEE 5th Innovative Smart Grid Technologies Conference, Washington, DC*, 2014.

[5] F. Milano and R. Zárate-Miñano, "A Systematic Method to Model Power Systems as Stochastic Differential Algebraic Equations," *IEEE Transactions on Power Systems*, vol. 28, no. 4, pp. 4537–4544, Nov. 2013.

[6] K. Wang and M. Crow, "Numerical simulation of stochastic differential algebraic equations for power system transient stability with random loads," in *Power and Energy Society General Meeting, 2011 IEEE*, July 2011, pp. 1–8.

[7] Z. Y. Dong, J. H. Zhao, and D. Hill, "Numerical simulation for stochastic transient stability assessment," *Power Systems, IEEE Transactions on*, vol. 27, no. 4, pp. 1741–1749, Nov 2012.

[8] R. Zárate-Miñano, M. Anghel, and F. Milano, "Continuous Wind Speed Models based on Stochastic Differential Equations," *Applied Energy*, vol. 104, pp. 42–49, Apr. 2013.

[9] J. Rosenlind, F. Edstrom, P. Hilber, and L. Soder, "Modeling Impact of Cold Load Pickup on Transformer Aging Using Ornstein-Uhlenbeck Process," in *IEEE PES General Meeting*, Jul. 2013, pp. 1–1.

[10] R. Weron, B. Kozlowska, and J. Nowicka-Zagrajek, "Modeling electricity loads in California: a continuous-time approach," *Physica A: Statistical Mechanics and its Applications*, vol. 299, no. 1-2, pp. 344–350, Oct. 2001.

[11] M. Perninge, M. Amelin, and V. Knazkins, "Load modeling using the Ornstein-Uhlenbeck process," in *Power and Energy Conference, 2008. PECon 2008. IEEE 2nd International*, Dec. 2008, pp. 819–821.

[12] P. Tankov, *Financial modelling with jump processes*. CRC press, 2003, vol. 2.

[13] F. E. Benth and J. Šaltytė-Benth, "Stochastic modelling of temperature variations with a view towards weather derivatives," *Applied Mathematical Finance*, vol. 12, no. 1, pp. 53–85, 2005.

[14] B. M. Bibby and M. Sørensen, "Hyperbolic processes in finance," *Handbook of heavy tailed distributions in finance*, pp. 211–248, 2003.

[15] B. M. Bibby and M. Srensen, "Martingale Estimation Functions for Discretely Observed Diffusion Processes," *Bernoulli*, vol. 1, no. 1/2, pp. 17–39, Mar. 1995.

[16] P. Alaton, B. Djehiche, and D. Stillberger, "On modelling and pricing weather derivatives," *Applied Mathematical Finance*, vol. 9, no. 1, pp. 1–20, Mar. 2002.

[17] R. F. Fox, "Numerical simulations of stochastic differential equations," *Journal of statistical physics*, vol. 54, no. 5-6, pp. 1353–1366, 1989.

[18] D. Higham., "An Algorithmic Introduction to Numerical Simulation of Stochastic Differential Equations," *SIAM Rev.*, vol. 43, no. 3, pp. 525–546, Jan. 2001.

[19] T. H. Rydberg, "The normal inverse gaussian lévy process: simulation and approximation," *Communications in statistics. Stochastic models*, vol. 13, no. 4, pp. 887–910, 1997.

[20] W. Breymann and D. Lüthi, "ghyp: A package on generalized hyperbolic distributions," *Manual for R Package ghyp*, 2013.

[21] R. Weron, M. Bierbrauer, and S. Trück, "Modeling electricity prices: jump diffusion and regime switching," *Physica A: Statistical Mechanics and its Applications*, vol. 336, no. 1, pp. 39–48, 2004.

[22] A. Cartea and M. G. Figueroa, "Pricing in electricity markets: a mean reverting jump diffusion model with seasonality," *Applied Mathematical Finance*, vol. 12, no. 4, pp. 313–335, 2005.