On the Impact of Data-Driven Stochastic Load Models on Power System Dynamics

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***Abstract*—This paper analyzes the impact on the dynamic behavior of power systems of modeling stochastic processes of the load consumption through different probability density functions (PDFs). The statistical properties of the processes modeled on the loads are obtained through the load consumption data measured using micro-synchrophasors (μPMU's). Simulation results show that stochastic processes with different PDFs, though similar in statistical properties, lead to different dynamic behaviors. The case study also demonstrates that the stochastic processes affect different power system quantities differently.**

***Index Terms*—Stochastic processes, probability distribution function, standard deviation, volatility, transient stability, load consumption.**

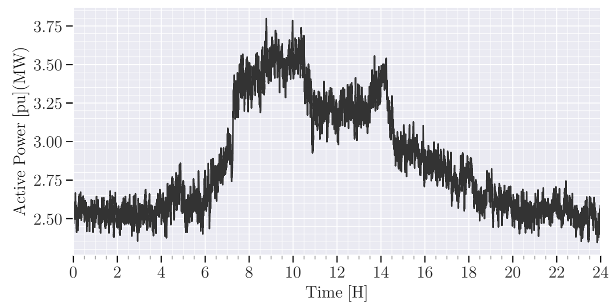

Fig. 1: Active power consumption in pu with a base of 100 MVA [5].

I. Introduction

A. Motivation

Volatility is defined as fast variations around a mean value in the time scale of power system dynamic. In power systems, volatility can be effectively modeled as a stochastic process using stochastic differential equations (SDEs) [1]. For the stochastic process to be realistic, it has to properly capture statistical properties, such as the autocorrelation [2], correlation [3], and the probability distribution function (PDF) [4]. The motivation of this paper is the observation that stochastic processes with different PDFs can lead to different impacts on the dynamic behavior of the power system [4]. This paper provides a discussion on the impact of modeling stochastic loads through various stochastic processes on power system dynamic response. The study is data-driven and based on measurement time series such as the one shown in Fig. 1.

B. Literature Review

The impact of load models on the power system dynamic has been analyzed in a number of studies such as [6]–[8]. The drawback of these works is that they do not include volatility in load models. The recent studies that explore the impact of load volatility on the power system dynamic are [2], [3], [9]. These works model power system dynamic via stochastic differential-algebraic equatuions (SDAEs), which allows for the inclusion of stochastic processes in power systems. Even though these works utilize various stochastic processes to model sotocahstic loads and are able to model the processes with different PDFs, they do not analyze the impact of processes modeled with similar statistical properties but different PDFs on the dynamic of power system.

The impact of processes with different statiscal properties but similar PDFs using SDAEs was first analyzed in [4]. This reference shows that, when wind volatility is modeled with different PDFs exhibiting similar statistical properties, the power system dynamic undergoes different behaviors. It also demonstrates that certain wind speed PDFs worsen the impact of contingencies. The work in [4] is the main motivation for the study presented in this paper.

C. Contributions

This work discusses the impact of data-driven stochastic load models on the dynamic response of power systems. It is shown that the load consumption modeled using different stochastic processes exhibiting similar statistical behavior have distinct impacts on the power system quantities. The case study shows that different power system variables are affected differently. The study is founded on the real-world data described in [5], where different stochastic load models were derived based on the measurements obtained with micro-synchrophasors (μPMU's) [10].

D. Organization

The remainder of the paper is organized as follows. Section II-A introduces SDAE model that is utilized to introduce volatility in power system dynamic. Section II-B outlines the stochastic load model. The load measurement data to determine the statistical properties and shape of the PDFs is analyzed in Section III-A. Section III-B introduces SDE models to formulate the processes with various PDFs. Section

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IV illustrates a case study utilizing a 9-bus system with the inclusion of stochastic loads and analyzes the impact of various stochastic processes on the power system dynamic response. Finally, Section V draws conclusions.

II. Modeling

A. Stochastic Differential-Algebraic Equations

Stochastic processes can be included into the dynamic model of power systems through the following set of $(l + m + n)$-dimensional SDAEs [1], [5]:

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{\eta}) \,, \tag{1}$$

$$\boldsymbol{0} = \boldsymbol{g}(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{\eta}) \,, \tag{2}$$

$$\dot{\boldsymbol{\eta}} = \boldsymbol{a}(\boldsymbol{\eta}) + \boldsymbol{b}(\boldsymbol{\eta}) \odot \boldsymbol{L} \,, \tag{3}$$

where $\boldsymbol{f} : \mathbb{R}^{l+m+n} \mapsto \mathbb{R}^m$, and $\boldsymbol{g} : \mathbb{R}^{l+m+n} \mapsto \mathbb{R}^l$ are the differential and algebraic equations; $\boldsymbol{x} \in \mathbb{R}^l$ is the vector of state variables; $\boldsymbol{y} \in \mathbb{R}^m$ is the vector of algebraic variables; $\boldsymbol{\eta} \in \mathbb{R}^n$ are the stochastic processes; $\boldsymbol{a} \in \mathbb{R}^n$ is the vector of *drift* term; $\boldsymbol{b} \in \mathbb{R}^n$ is the vector of *diffusion* term; $\odot$ represents the Hadamard product, i.e., the element-by-element product of two vectors; and $\boldsymbol{L} \in \mathbb{R}^n$ is a vector of n-dimensional Lévy processes.

Each element of $\boldsymbol{L}(t)$, say $L_i(t), i = 1, ..., n$, is a Lévy process if it satisfies the following conditions [11]:

1) $L_i(0) = 0$, with probability 1.
2) $L_i(t)$ has independent increments, i.e., $\forall 0 \leq s < t$, $\text{cov}[dL_i(t), dL_i(s)] = 0$.
3) $L_i(t)$ has stationary increments, i.e., the probability distribution of the increments $dL_i(t)$ and $dL_i(s)$ depends only on the length of the time interval $t - s$.
4) $L_i(t)$ has stochastic continuity, i.e., $\forall \epsilon > 0$, $\lim_{h \to 0} P(|X_{t+h} - X_t| \geq \epsilon) = 0$.

Note that Wiener process and Poisson process are examples of the Lévy processes.

Equations (3) are SDEs that can be formally integrated to obtain:

$$\boldsymbol{\eta}(t) = \boldsymbol{\eta}(t_o) + \int_{t_o}^{t} \boldsymbol{a}(\boldsymbol{\eta}(s))dt + \int_{t_o}^{t} \boldsymbol{b}(\boldsymbol{\eta}(s))d\boldsymbol{L}(s) \,, \tag{4}$$

In (4), the drift term is being integrated w.r.t. time, and can be represented as a traditional integral such as the Riemann-Stieltjes integral. However this is not the case for the diffusion term which is being integrated w.r.t. the Lévy process, which is a stochastic process. The Lévy process has unbounded variations, which makes the integral non-deterministic. In power systems, the stochastic integral is usually represented as an Itô's integral, and is integrated using Itô's calculus [12], [13].

B. Load Consumption

In this paper, we utilise a well-known voltage dependent stochastic load model that has been used in various power system studies [1], [6], and is written as:

$$\begin{aligned} p_L(t) &= (p_{L0} + \eta_p(t))(v(t)/v_0)^k \\ q_L(t) &= (q_{L0} + \eta_q(t))(v(t)/v_0)^k \,, \end{aligned} \tag{5}$$

where $p_L(t)$ and $q_L(t)$ are the active and reactive powers of the load, respectively; and p_{L0} and q_{L0} are the active and reactive powers of load at $t = 0$; $v(t)$ is the voltage magnitude at the load bus; v_0 is the value of this voltage magnitude at $t = 0$; the coefficient k defines the voltage dependence of the load such that $k = 0$ imposes constant power loads while $k = 2$ makes it a constant impedance load; and $\eta_p(t)$ and $\eta_q(t)$ are the stochastic processes associated to active and reactive load powers and are modeled using (3).

III. Data Modeling

A. Data Analysis

In this section, the measurement data of the active power is analyzed to determine the appropriate stochastic process to be modeled on the load consumption. The stochastic process in (3) has three parameters drift a, diffusion b and the driving process L. All are discussed as follows.

The drift term in (3) is responsible for defining the time evolution of the process. The drift of a stochastic process can be conveniently determined by calculating the Autocorrelation Function (ACF). The ACF measures the correlation of a time series with the delayed version of itself. ACF can be mathematically expressed as a function of time lag τ, and written as:

$$R(\tau) = \frac{E[(\eta_t - \mu)(\eta_{t+\tau} - \mu)]}{\sigma^2} \,, \tag{6}$$

where E is the expectation operator; η_t is the value of the process at time t; and σ is the standard deviation of process η. Off course the ACF at $\tau = 0$ is $R(0) = 1$.

The ACF of the active power measurements for a period of 15 minutes is illustrated in Fig. 2. The choice of 15 minutes is motivated by the fact that the process becomes stationary at 15 minutes. This point is clarified later in this section. The exponential function fitting the data ACF is also shown in Fig. 2. It can be observed that the ACF follows the exponentially decaying function.

For this reason, we model the drift term as an exponentially decaying process and rewrite (3) as:

$$\dot{\boldsymbol{\eta}} = -\boldsymbol{\alpha} \odot (\boldsymbol{\eta} - \boldsymbol{\mu}) + \boldsymbol{b}(\boldsymbol{\eta}) \odot \boldsymbol{L} \,, \tag{7}$$

where $\boldsymbol{\alpha} \in \mathbb{R}^n$ is the vector of autocorrelation coefficients or the mean-reversion speeds; and $\boldsymbol{\mu} \in \mathbb{R}^n$ is the mean value of the process $\boldsymbol{\eta}$. The rest of the variables have the same meaning as in (3).

Next, we determine the diffusion and the Lévy process. The diffusion term in combination with the Lévy process

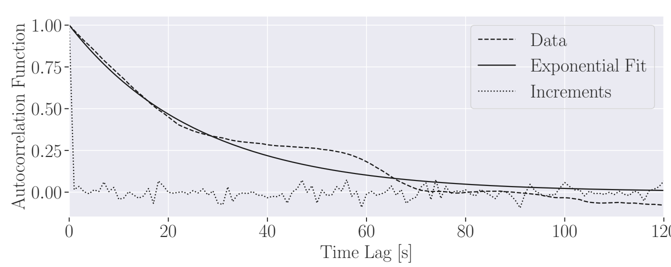

Fig. 2: Autocorrelation function (ACF) of active power.

determines the PDF of the stochastic process η. Whereas the Lévy process provides increments for the SDE in (3).

To model the driving process, i.e., the Lévy process, the distribution of the active power increments Δp is observed. Two conditions are to be fulfilled here, i.e., independence and stationarity. The independence of the Δp is demonstrated by illustrating the ACF in Fig. 2. This shows that no self correlation is observed in Δp, and hence the increments are independent.

Next step is to demonstrate that the increments follow stationary distribution. Figure 3 shows standard deviation $\sigma(\Delta p)$ and mean $\mu(\Delta p)$ of the increments of the active power for a period of 60 minute with a rolling window of 15 minute. The size of the window is chosen to be 15 minutes because $\sigma(\Delta p)$ and $\mu(\Delta p)$ become stationary at 15 minutes.

Figure 3 shows that $\sigma(\Delta p)$ settles at different values for different windows. This is conformed in Fig. 4, which shows the σ of Δp and α of the active power calculated from the data for a rolling window of 15 minutes. It can be observed that at the maximum value of σ the value of α is the lowest. While the maximum α corresponds to a value of σ that is not the maximum. This result is very important for determining the impact of stochastic loads on the power system dynamics. In fact, reference [2] shows that high values of α coupled with higher σ are responsible for causing instabilities in the system.

Finally, we determine the fitting PDF for the stochastic process. At first glance, the Δp seems to be normally distributed as can be seen in Fig. 5. With careful inspection of the PDF, it is easily determined that the Gaussian distribution fails to capture the long tails and the narrow shape of Δp. This shows that modeling Δp with the Gaussian distribution does not merit. However, the Gaussian distributed process might be the best candidate to approximate the behavior of the actual process in cases where the information on the measured PDF is not available.

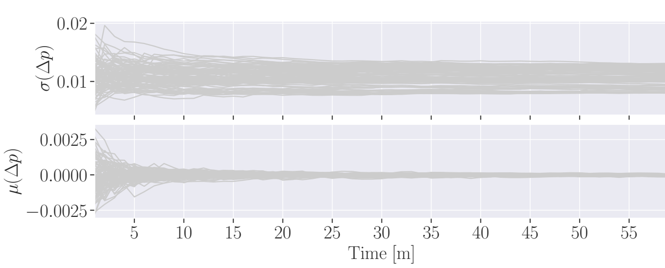

Fig. 3: Standard deviation (σ) and mean (μ) of the increments Δp for rolling window of 15 minutes.

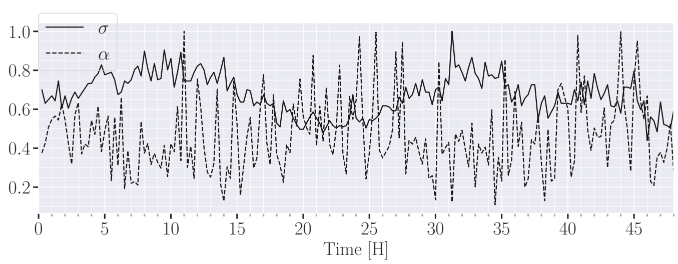

Fig. 4: Standard deviation (σ) and autocorrelation coefficient (α) for rolling window of 15 minutes. Note that the curves are normalized with $\max\{\sigma\} = 0.0157$ pu and $\max\{\alpha\} = 0.045\ \mathrm{s}^{-1}$.

A more representative distribution that exhibits symmetry and can capture long tails is the Normal Inverse Gaussian distribution (NIG). If X_{NIG} is a NIG distributed stochastic process then:

$$X_{\mathrm{NIG}} \sim \mathrm{NIG}(\kappa, \beta, \delta, \mu) \tag{8}$$

where κ is a steepness parameter, β is an asymmetry parameter, δ is a scale and μ is the location parameter. As noted above in this section, a symmetrical distribution is required. For this reason, $\beta = 0$ and $\mu = 0$ are assumed. These values lead to a symmetrical distribution centered on a zero mean. Note that a NIG process is not so straight-forward to implement in software as it is to implement Gaussian processes.

A further improvement to ease the implementation of the stochastic process is required. This is provided by a compound process a jump-diffusion process. This process was utilized in [5] to capture the long tails of Δp. It is a combination of the Gaussian and the compound Poisson process. Note that in this paper, we are interested in evaluating the impact of various stochastic processes on the dynamic of the power system. The procedure to fit the distribution is not discussed. Interested reader can find details on the topic in [5].

B. Process Modeling

The SDE in (7) can be used to achieve different distributions discussed Section III-A. To elaborate on this, we start by describing the Lévy as a standard Wiener process $\boldsymbol{W} \in \mathbb{R}^n$, and re-write (7) as:

$$\dot{\boldsymbol{\eta}}_{\mathrm{OU}} = -\boldsymbol{\alpha} \odot (\boldsymbol{\eta} - \boldsymbol{\mu}) + \boldsymbol{\beta} \odot d\boldsymbol{W}\,, \tag{9}$$

where $\boldsymbol{\beta} \in \mathbb{R}^n$ defines the standard deviation of the resulting process such that $\beta_i = \sqrt{2\alpha_i}\,\sigma_i$; and $\boldsymbol{\eta}_{\mathrm{OU}}$ is a Gaussian distributed process with mean $\boldsymbol{\mu}$ and standard deviation $\boldsymbol{\sigma}$. The process in (9) is known as Ornstein-Uhlenbeck's process (OUP).

Next, the NIG distributed process model is required. As explained in Section III-A, it is complicated to implement such a distribution in programming. In this paper, a more convenient method is utilized to simulate the NIG distributed process. The NIG distributed process can be conveniently obtained as a memoryless transformation of the OUP in (9). This is achieved by calculating the inverse Cumulative Density Function (CDF)

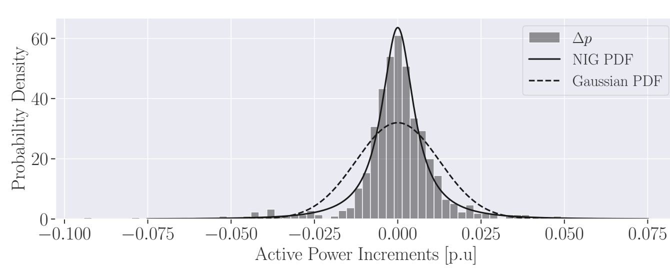

Fig. 5: Increments of active power. Data is converted to per unit with a base of 100 MVA.

of the Gaussian CDF of the OUP. This is mathematically expressed as:

$$\eta_{\mathrm{NIG}}(t) = F^{-1}(\Phi(\eta_{\mathrm{OU}}(t)) \,, \tag{10}$$

where η_{NIG} is a stochastic process that follows NIG distribution; F^{-1} is the inverse CDF of NIG; and Φ is the CDF of the Gaussian distribution.

Finally, the jump-diffusion process is conveniently obtained by introducing jumps in the OUP, and writing (9) as:

$$\dot{\boldsymbol{\eta}} = -\boldsymbol{\alpha} \odot (\boldsymbol{\eta} - \boldsymbol{\mu}) + \boldsymbol{\beta} \odot d\boldsymbol{W} + \boldsymbol{c}(\boldsymbol{\eta}) \odot d\boldsymbol{J}(t) \,, \tag{11}$$

where $\boldsymbol{c} \in \mathbb{R}^n$ defines the amplitude of the jumps; and $\boldsymbol{J} \in \mathbb{R}^n$ is a Poisson random variable with intensity λ such that

$$dJ_i = \begin{cases} 1 & \text{with probability } \lambda dt \\ 0 & \text{with probability } 1 - \lambda dt \end{cases} \tag{12}$$

IV. Case Study

This section analyzes the impact of stochastic load models, which are based on measurement data, on the dynamic behavior of the power system. The power system chosen for the study is the 9-bus Western System Coordinating Council (WSCC), which contains 3 synchronous generators, 3 load devices and 9 buses. The WSCC is also equipped with turbine governors, an automatic generation control and automatic voltage regulators to avoid any unrealistic instabilities. All simulation results presented in this section were obtained with the software tool Dome [14]. SDAEs are integrated using the trapezoidal method for the drift and the Euler-Maruyama integration scheme for the diffusion term [12], [15] as well as jumps [16].

The stochastic processes are introduced into the WSCC through the active and reactive powers on the load consumption, which are modeled via stochastic load model in (5) using constant power loads. The processes η_p and η_q in (5) are modeled through the Gaussian, NIG, and jump-diffusion processes. For the evaluation of the impact of stochastic processes and their statistical properties on the power system dynamic, two scenarios are proposed as follows:

- S1: simulates η_p and η_q with highest values of α calculated from the measured active and reactive power.
- S2: simulates η_p and η_q with lowest values of α calculated from the measured active and reactive power.

Scenario S1 has the following parameters: $\alpha_p = 0.045\mathrm{s}^{-1}$; $\alpha_q = 0.1115\mathrm{s}^{-1}$; $\sigma_p = 0.0117$ pu and $\sigma_q = 0.01435$ pu for Gaussian distribution; and $\kappa_p = 100.11$, $\delta_p = 0.0141$, $\kappa_q = 37.70$ and $\delta_q = 0.00753$ for NIG. Scenario S2 has the following parameters: $\alpha_p = 0.0048\mathrm{s}^{-1}$; $\alpha_q = 0.0041\mathrm{s}^{-1}$; $\sigma_p = 0.0119$ pu and $\sigma_q = 0.0196$ pu for Gaussian distribution; and $\kappa_p = 64.51$, $\delta_p = 0.009$, $\kappa_q = 150.34$ and $\delta_q = 0.0575$ for NIG.

First, the PDFs of the load measurement data and η_p for scenarios S1 and S2 are shown in Fig. 6. From Fig. 6, it is clear that the NIG and the jump-diffusion processes are good candidates for capturing the long tails of the measurement data. Note that even though both the NIG and the jump-diffusion processes capture the tails with good precision, only NIG process can capture the height and shape of the PDF of the load measurement data.

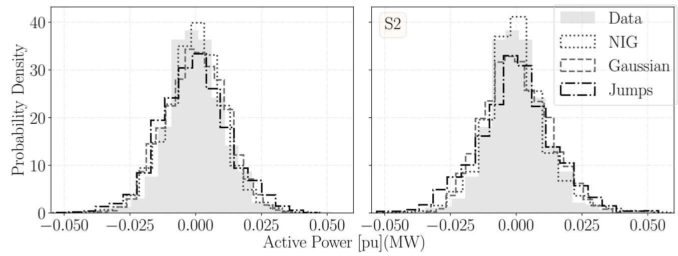

Fig. 6: PDF of load measurement data and η_p for scenarios S1 and S2.

Next, the ACF of η_p and the load measurement data for scenarios S1 and S2 is shown in Fig. 7. Figure 7 clearly shows that ACFs of η obtained from all the PDFs are able to follow the ACF of the load measurement data. Note that ACF in both the scenarios take different times to reach an equilibrium point. These differences in time account for the slow and fast time-varying processes. As demonstrated in a previous study [2], fast time-varying processes coupled with high standard deviations can cause instabilities in the power system.

Next, the bus voltage magnitude at the load bus 6 v(Bus 6) for scenarios S1 and S2 is shown in Fig. 8. Note the difference in the spread of the PDFs for scenario S1 versus S2. It is clear that S1 has a larger spread as compared to S2. This is due to the fact that S1 has a higher α as compared to S2.

Another notable result is the spread of the PDFs for each scenario. Figure 8 illustrates that the Gaussian and the jump-diffusion processes follow similar behavior. Also the tails of these two processes are quite close to each other. On the other hand, the NIG process shows a bigger spread and longer tails than the Gaussian and the jump-diffusion processes.

Interestingly, the spread and tails of the PDFs of the bus voltages for the NIG case strongly depends on the location of the bus. For example, Fig. 9 shows the PDFs of v at bus 2 for both scenarios. Compared to Fig. 8, for scenario S2, the NIG case shows a voltage with a significantly different behavior, namely, narrower tails. NIG, Gaussian and jump-diffusion processes have similar effect on the voltage at bus 2. This result is non-intuitive. However, it can be explained considering that the voltage at bus 2 is controlled by the AVR of the synchronous machine connected at that bus, whereas bus 6 is an uncontrolled load bus.

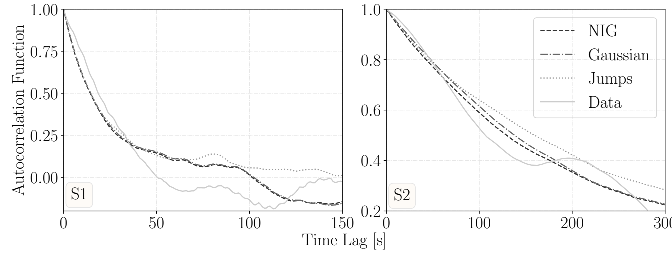

Fig. 7: Autocorrelation function of load measurement data and η_p for scenarios S1 and S2.

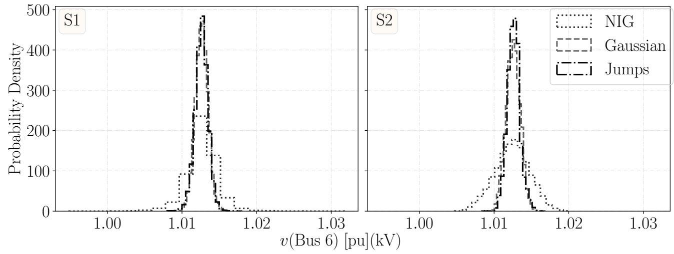

Fig. 8: PDF of bus voltage magnitude at the load bus 6 for S1 and S2.

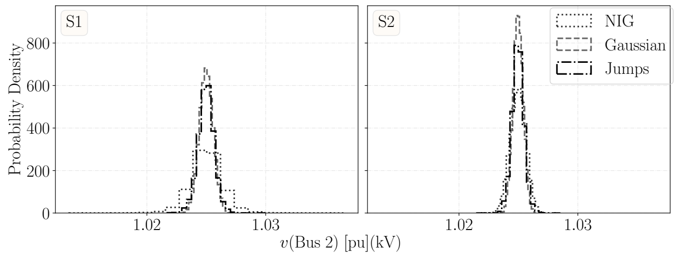

Fig. 9: PDF of bus voltage magnitude at the generator bus 2 for S1 and S2.

Next, the rotor speed and the active power injection of the synchronous machine G1, ω(G1) and p_g(G1) respectively, are observed. Figures 10 and 11 illustrate the PDFs of ω(G1) and p_g(G1) for both scenarios. The intuitive result in the both Figs. 10 and 11 is that scenario S1 has a larger spread of the process as compared to S2. It is also obvious from Figs. 10 and 11 that the Gaussian and NIG processes have overlapping PDFs while the jump-diffusion process has longer tails.

Simulation results indicate that the dynamic of the power system is affected by the modeling PDF of the stochastic process. This is consistent with the study presented in [4]. Moreover, this paper also shows that the behavior of particular power system quantity also depends on the location of the noise, on the network topology and on the control.

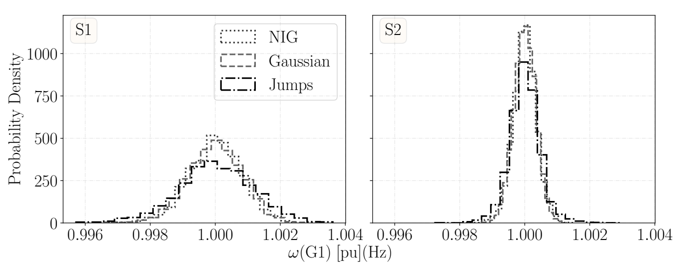

Fig. 10: Probability density of the rotor speed of the synchronous machine G1 ω(G1) for scenarios S1 and S2.

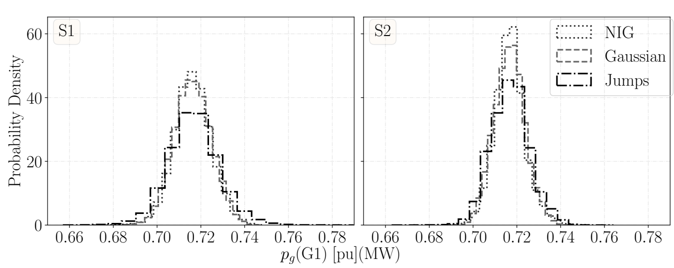

Fig. 11: Probability density of the the active power injection of the synchronous machine G1 p_g(G1) for scenarios S1 and S2.

V. Conclusions

This paper analyzes the impact of data-driven stochastic load models on the dynamic response of power systems. With this aim, the paper focuses on the effect of long tails in the PDF of stochastic processes such as load consumptions.

The case study demonstrates that stochastic processes with PDFs having similar statistical properties but different functions can cause over- or under-voltages and/or large frequency variations in the power system. In particular, Gaussian processes, Gaussian processes with jump-diffusion and NIG processes are compared. Among these three, NIG process is the one that impacts the most the voltage magnitudes and can cause the voltages to violate the system limits. On the other hand, the jump-diffusion process impacts mostly on the rotor-speed and active power injections of the machines.

Future work will investigate the differences in the behavior of the voltage, frequency and power injections based on the location of noise, network topology, load model and different PDFs.

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