Modeling Correlation of Active and Reactive Power of Loads for Short-Term Analysis of Power Systems

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***Abstract*—This paper discusses stochastic load modeling for dynamic analysis of power systems. The proposed approach enables modeling loads with correlated active and reactive powers. In particular, the proposed model is able to accurately reproduce the statistical properties of both stochastic variations and jumps of loads. The case study shows that, the correlation between the active and reactive powers consumed by loads impacts the level of uncertainty and stability limits of the system.**

***Index Terms*—Correlation, load modeling, Poisson process, stochastic differential equation, Wiener process**

I. Introduction

A. Motivation

Load power consumption is not fully deterministic. This has led to the proposal of a variety of stochastic load models in the literature. These models consider mostly long-term operation and forecast problems [1], [2]. A modeling aspect that has not been adequately addressed is the statistical correlation in short-term dynamics of the active and reactive powers consumed by each load. This paper aims to address this aspect.

B. Literature Review

In power system stability studies the loads have traditionally been modeled as deterministic using either static or dynamic models [3], [4]. However, loads often vary in an uncertain manner in the short-term. Recent relevant studies that consider such an uncertainty of loads are as follows.

In [5], the effect of load uncertainty on voltage stability is considered using trajectory sensitivity analysis. Other such probabilistic analysis considering the effect of load uncertainty on the system stability are presented in [6], [7]. In [8] it is shown that important information is lost when only probabilistic analysis of the system is considered for stability analysis. For this reason it is important to model the load uncertainty as a stochastic processes evolving with time.

Guðrún Margrét Jónsdóttir and Federico Milano are funded by Science Foundation Ireland, under Investigator Programme, Grant No. SFI/15/IA/3074.

Load uncertainty can be modeled as a stochastic process in time using Stochastic Differential Equations (SDEs). SDEs are continuous with respect to time and are therefore well equipped to reproduce transient random fluctuations. Since power system models are typically formalized as a set of Differential-Algebraic Equations (DAEs), SDEs can be readily incorporated into the system model. The resulting model is a set of Stochastic Differential-Algebraic Equations (SDAEs).

Several studies have used SDEs for load modeling. A modified exponential recovery model, with a purely diffusion term SDEs, modeling the load uncertainty is utilized in [9]. In [10]–[12], Ornstein-Uhlenbeck (OU) SDE processes are used to model the load variations and in [13] OU processes including jumps are proposed for load modeling. However, none of these stochastic models consider the correlation in the active and reactive power.

Figure 1 shows an example of active and reactive power measurements obtained with a PMU. The data indicate that there is a clear correlation between the stochastic component and the jump component when it comes to jump time and size in the measured active and reactive power. Thus, the active and reactive power of the load are, at least in some cases, correlated and their correlation needs to be modeled.

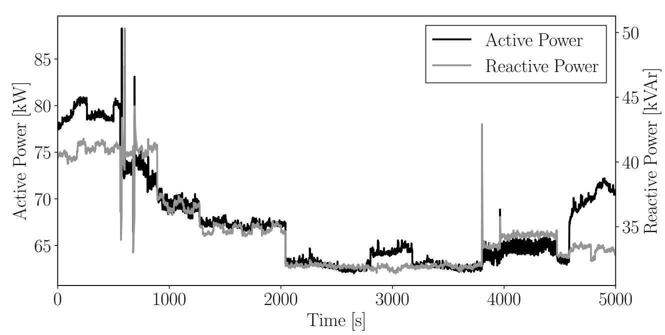

Fig. 1. Active and reactive power consumption of a load measured with a PMU.

C. Contributions

This paper improves the OU load model with jumps presented in [13]. The result is a model that is able to take into account the correlation in the active and reactive powers of the load consumption. Both correlation in the stochastic component and the jump component, when it comes to the jump time and the jump size, can be modeled with the proposed model. A case study based on the 9-bus 3-machine test system, including the proposed load model, is used to demonstrate the effect of modeling the correlation on system dynamics.

The specific contributions of this paper are twofold.

- A mathematical approach based on SDEs that allows modeling the correlation between the active and reactive power consumption.
- An appraisal of the effect that correlation between the active and reactive load powers has on the dynamics of power systems.

D. Organization

The remainder of the paper is organized as follows. Section II presents the proposed correlated load model. A case study considering the 9-bus 3-machine system is presented in Section III. There the effect of modeling the correlation in the active and reactive power of the loads on the dynamic analysis of the test system is studied. Finally, in Section IV conclusions are drawn and future work is outlined.

II. Stochastic Modeling of Loads

This section presents the proposed stochastic load model. The model is based on the SDE-based jump diffusion model proposed in [13]. In this paper the model in [13] is modified to include correlation in the active and reactive power component of the load.

In Subsection II-A a brief introduction to jump diffusion SDEs is presented. The modeling of the correlation in the stochastic and jump component of the SDE-based model is outlined in Subsection II-B. The jump diffusion model in Subsection II-A is then coupled with the correlation procedure presented in Subsection II-B to model the stochastic variations of the load as presented in Subsection II-C.

A. SDE Jump Diffusion Model

Stochastic Differential Equations (SDEs) are a prominent mathematical modeling technique and have been utilized in previous power systems studies, e.g for modeling loads [10]–[12], wind [14], [15] and tidal generation [16]. Additionally, SDEs with jumps have been used to model solar [17], [18] as well as loads [13].

A general one-dimensional jump diffusion model, i.e. SDE with jumps is defined as:

$$d\eta_J(t) = a(t,\eta_J)dt + b(t,\eta_J)dW(t) + c(t,\eta_J)dJ(t). \quad (1)$$

where a and b are continuous functions and are referred to as the drift and diffusion term of the SDE, respectively. $\{W(t), t > 0\}$ is the Wiener processes, which is a random function characterized by the following properties:

1) $W(0) = 0$, with probability 1.
2) The function $t \mapsto W(t)$ is continuous in t.
3) If $t_1 \neq t_2$, then $W(t_1)$ and $W(t_2)$ are independent.
4) For $\forall t_i \geq 0$, all increments, $\Delta W_i = W(t_{i+1}) - W(t_i)$, are normally distributed, with mean 0 and variance $h = t_{i+1} - t_i$, i.e., $\Delta W_i \sim \mathcal{N}(0, h)$.

The jump part is modeled through c, the jump coefficient which determines the jumps size and $\{J(t), t > 0\}$ which is the Poisson process. The Poisson process with intensity $\lambda > 0$ is defined as a type of stochastic process called a counting process which is characterized by the following properties:

1) $J(0) = 0$, with probability 1.
2) It has stationary independent increments.
3) The number of events (or points) in any given time interval of length t is a Poisson random variable with the mean λt.
4) Its probability density function is:

$$f(x, \lambda) = \frac{\lambda^x}{x!}\exp(-\lambda). \quad (2)$$

An in-depth discussion on SDEs is outside the scope of this paper. The interested reader is referred to [19] for details on SDE theory and numerical methods.

B. Modeling Correlation

Loads are modeled as two variables, namely the the active and reactive power consumed. The stochastic jump diffusion model, presented in (1) enables the modeling of these two quantities independently. However, the active and reactive power are linked and thus may be correlated.

This subsection presents the modeling procedure used to define the correlation of two SDEs. The resulting model consists in two correlated Wiener components and two correlated Poisson jump components.

1) Correlated Wiener Processes: The Wiener process is the stochastic driving components of SDEs. The correlation of two Wiener processes, say $W_1(t)$ and $W_2(t)$, can be achieved by defining a third Wiener process, say $V(t)$, which is independent from $W_1(t)$. The correlation between $W_1(t)$ and $W_2(t)$ is defined through a parameter $\rho_W(t)$. This can be constant or time varying. In the latter case, $\rho_W(t)$ is modeled as another stochastic process. In this work, ρ_W is assumed to be constant as in [20]. Then $W_2(t)$ is defined as the following adapted Wiener process:

$$dW_2(t) = \rho_W dW_1(t) + \sqrt{1-\rho_W^2}\,dV(t). \quad (3)$$

2) Correlated Poisson Jump Processes: The jumps of two jump diffusion SDEs as presented in (1) can also be correlated. To model the correlation between two Poisson distributed jumps, $J_1(t)$ and $J_2(t)$, three Poisson distributed jumps $n_1(t)$, $n_2(t)$ and $n_3(t)$, with jump rates λ_1, λ_2 and λ_3 are used [21]. $J_1(t)$ and $J_2(t)$ are defined as:

$$J_1(t) = n_1(t) + n_3(t), \tag{4}$$
$$J_2(t) = n_2(t) + n_3(t). \tag{5}$$

Thus, the mean of the two Poisson jump processes is:

$$(\lambda_i + \lambda_3)dt, \quad i = 1, 2 \tag{6}$$

Their covariance is λ_3 and their correlation is:

$$\rho_J = \frac{\lambda_3}{\sqrt{(\lambda_1 + \lambda_3)(\lambda_2 + \lambda_3)}}. \tag{7}$$

C. Proposed Load Model

The SDE based model, presented in [13], models the load variations using an Ornstein-Uhlenbeck SDE model with jumps. This load model is developed based on the well-known voltage dependent load model. However, it models the active and reactive power independently. The updated load model proposed in this paper utilizes the model proposed in [13] but enables the stochastic and jump component of the active and reactive power to be correlated as presented in Subsection II-B. Thus, the proposed load model is:

$$\begin{aligned} p_L(t) &= (p_{L0} + \eta_p(t))(v(t)/v_0)^k, \\ q_L(t) &= (q_{L0} + \eta_q(t))(v(t)/v_0)^k, \\ d\eta_p(t) &= \alpha_p(\mu_p - \eta_p(t))dt + \sigma_p dW_1(t) + \varsigma_p(t)dJ_1(t), \\ d\eta_q(t) &= \alpha_q(\mu_q - \eta_q(t))dt + \sigma_q dW_1(t) + \varsigma_q(t)dJ_2(t), \end{aligned} \tag{8}$$

where $p_L(t)$ and $q_L(t)$ are the active and reactive power of the load, respectively, and p_{L0} and q_{L0} are parameters representing active and reactive load powers at $t = 0$. $v(t)$ is the voltage magnitude at the bus where the load is connected and v_0 is the value of this voltage magnitude at $t = 0$.

The model in (8) can, through the exponent k define whether the load is a constant power load ($k = 0$), a constant current load ($k = 1$) or a constant impedance load ($k = 2$). The stochastic variability of the load is modeled through the stochastic processes $\eta_p(t)$ and $\eta_q(t)$ which are formulated as SDE jump diffusion processes, where α is the mean-reversion speed, μ is the mean and σ is the diffusion component of the process.

The jump amplitudes $\varsigma_p(t)$ and $\varsigma_q(t)$ are normally distributed random numbers, namely $\varsigma_p(t) \sim N(0, \sigma_{\varsigma_p})$ and $\varsigma_q(t) \sim N(0, \sigma_{\varsigma_q})$. They are modeled to have a correlation ρ_{J_s} in the same way as the Wiener process, as presented in Subsection II-B. The correlation in the Wiener ($W_1(t)$ and $W_2(t)$) and Poisson processes ($J_1(t)$ and $J_2(t)$) are modeled as presented in Subsection II-B.

III. Case Study

A. Test System

The test power system used in this case study is the Western System Coordinating Council (WSCC) 3-machine, 9-bus system shown in Fig. 2. The system base is 100 MVA and the system frequency is 60 Hz. Further details on the model can be found in [22]. Note that for this case study the voltage levels of the system have been lowered compared to the original case. This is done to demonstrate a case where the lower voltage limits might be reached.

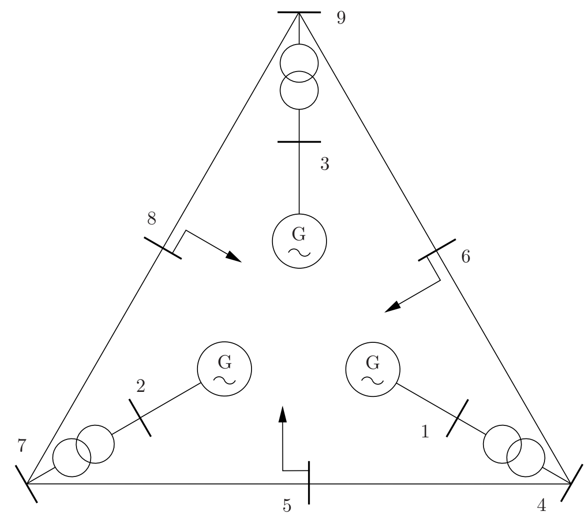

Fig. 2. The 3-machine, 9-bus test system.

The system is modeled as a set of SDAEs where the loads are modeled using SDEs as presented in (8). The parameters used for the load model are set as follows. The mean reversion parameters of the processes are set as $\alpha_p = \alpha_q = 0.02$. σ_p and σ_q are set individually for each load so that the standard deviation of the variations is 5% of p_{L0} and q_{L0}, respectively. The standard deviation of the jump size σ_{ς_p} and σ_{ς_q} is set to 10% of p_{L0} and q_{L0}, respectively.

All simulations are carried out using Dome, a Python-based software tool for power system analysis [23].

B. Simulation Results

For this case study two different cases are considered:

- *Case 1:* In this case the loads are modeled without jumps. That is $\varsigma_p(t) = \varsigma_q(t) = 0$. The effect of correlation in the stochastic component, through ρ_W, is studied.
- *Case 2:* In this case the loads are modeled with jumps. The effect of correlation in the jump time (ρ_J) and size (ρ_{J_s}) is studied.

Case 1: For this case three different scenarios are examined. These are: (a) $\rho_W = 0$, (b) $\rho_W = 0.5$ and (c) $\rho_W = 0.9$.

The test system is simulated $1,000$ times using Monte Carlo simulation for a duration of 110 seconds and with a time step of 0.05 seconds for both Cases 1.a and 1.c. A fault is applied in these simulations at Bus 7 at time 101 seconds that is cleared in 0.1 seconds. The resulting effect on the voltage at Bus 5 is shown in Figs. 3 and 4. In these figures, the black line presents the voltage mean value of the 1,000 simulations. This process gives an idea of the effect of the perturbation for the deterministic case, that is where the loads are not modeled as stochastic. In that case, the disturbance does not result in the voltage at Bus 5 reaching the lower voltage limit.

The results in Fig. 3 show that the voltage for only one trajectory goes below the voltage limit. This corresponds to 0.1% of the simulations. In Fig. 4 where the correlation in the active and reactive power of the loads is considered approximately 2% of the trajectories go below the limit and stay there for a longer duration. This shows the importance of considering the correlation in the active and reactive power uncertainty when the voltage stability of the system is considered.

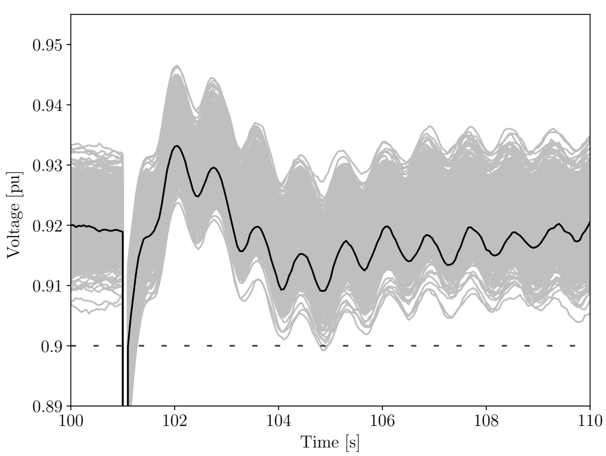

Fig. 3. Bus voltage at Bus 5 with no correlation (Case 1.a). The trajectories of the 1,000 simulations are shown in gray, the mean trajectory of the 1,000 simulations is shown in black and the lower voltage limit of 0.9 pu is highlighted with a dashed line.

The correlation also affects the steady state standard deviation of the voltage as can be seen in the spread of the simulated trajectories around the mean value in Fig. 4 when compared to Fig. 3. To further examine this point, the ramp rates of the voltage are computed and compared for the different scenarios. Ramp rates are computed as:

$$\Delta_h x_t = x_t - x_{t-h} \tag{9}$$

for time lags $h = 0-150$ s, where x_t is the voltage at time t. Then, the standard deviation of the ramp rates, $\Delta_h x_t$, for each time step h is computed.

For computing the ramp rates the system is simulated once for a duration of 10,000 seconds with a time step of 0.1 seconds without any disturbance. Figure 5 shows the standard deviations of the ramp rates of the voltage at Bus 5 for Case 1.a, 1.b and 1.c. The standard deviation increases as the correlation, ρ_W is increased. Thus, the more the stochastic variations of the active and reactive load power are correlated the bigger the variations/uncertainty in the voltages of the system.

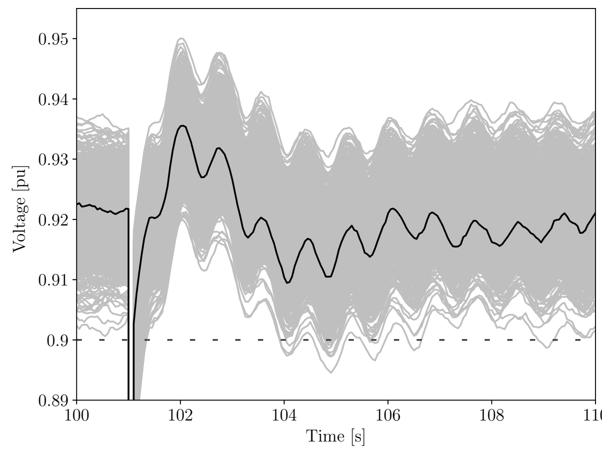

Fig. 4. Bus voltage at Bus 5 with correlation (Case 1.c). The trajectories of the 1,000 simulations are shown in gray, the mean trajectory of the 1,000 simulations is shown in black and the lower voltage limit of 0.9 pu is highlighted with a dashed line.

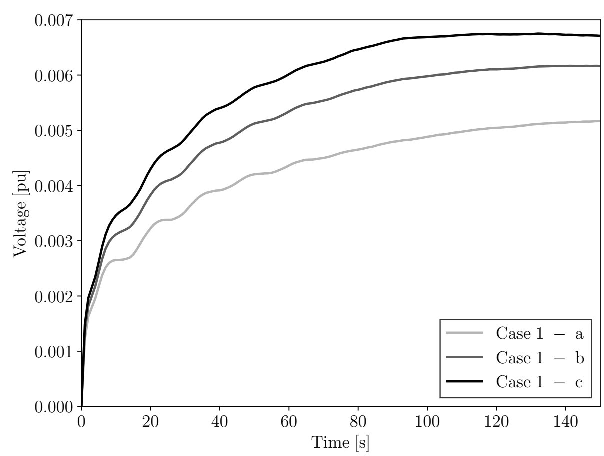

Fig. 5. The standard deviation of the ramps in the voltage at Bus 5 of the test system for Cases 1.a, 1.b and 1.c.

Case 2: In this case three different scenarios are considered. In all scenarios $\rho_W = 0$. These are (a) $\rho_J = \rho_{J_s} = 0$, (b) $\rho_J = 0.9$ and $\rho_{J_s} = 0$ and (c) $\rho_J = \rho_{J_s} = 0.9$.

Figure 5 compares the standard deviation of the ramp rates in the voltage at Bus 5 for Case 2. If the jump times are correlated (Case 2.b), the standard deviation of the ramps in the voltage is significantly decreased when compared to the base case with no correlation (Case 2.a). Case 2.b, where the jump times are correlated but not the jump size, results in the lowest levels of ramps in the voltage. Case 2.c, where the jump times and sizes are both correlated, results in lower voltage ramps than if no correlation is considered.

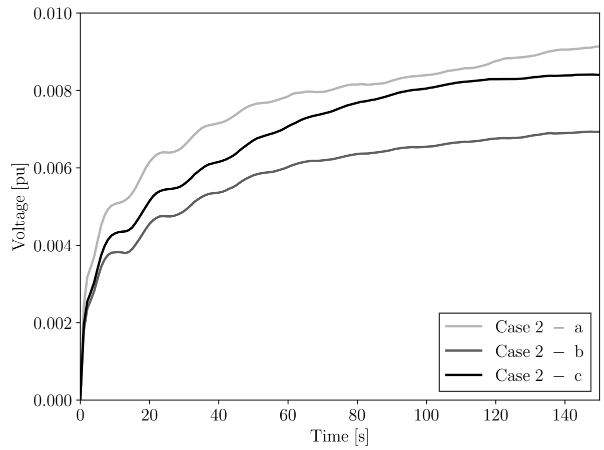

Fig. 6. The standard deviation of the ramps in the voltage at Bus 5 of the test system for Cases 2.a, 2.b and 2.c.

C. Remarks

The results for Cases 1 and 2 highlight the importance of modeling the correlation in the system loads as it affects the variations of the system voltages and thereby the stability limits of the system. The level and type of correlation in the active and reactive power may vary for different types of loads. Thus, it is important to examine measured data for different loads to be able to define this correlated behavior further.

IV. Conclusions

The paper deals with load modeling for dynamic analysis of power system. A SDE-based load model with jumps is proposed that can model correlation in the stochastic and jump component of the active and reactive power of loads.

A case study is presented where the effect of modeling the correlation in the load consumption is analyzed. It is shown that if the active and reactive powers of the load are correlated, the uncertainty of the system bus voltages is different compared to the scenario in which they are not correlated. Thus, it is important to define the level and type of correlation within the loads for accurate power system stability analysis.

In future work, measured load data will be analyzed to define how the correlation in the active and reactive powers may vary for different types of loads.

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