Comparison of Bus Frequency Estimators for Power System Transient Stability Analysis

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***Abstract*—The paper presents a study on the dynamic response of power system frequency control devices considering different approaches to estimate bus frequencies in power system simulators. The frequency signals considered in this paper are obtained based on the center of inertia, a commonly-used washout filter that approximates the derivative of the bus voltage phase angle, and a frequency divider formula developed by the authors. The dynamic behavior of frequency control devices such as thermostatically regulated loads is compared considering the three signals above. Different scenarios are discussed based on the IEEE 14-bus and New England 39-bus, 10-machine test systems.**

I. Introduction

In recent years, the regulation of the frequency based on non-synchronous devices has become more and more important. The penetration of distributed generation connected to the grid through power electronic converters such as VSC devices has led to the need to define proper primary frequency regulation for such devices [1]–[4]. Microgrids, flexible loads and energy storage devices are also expected to participate to the frequency regulation of the grid in the near future [5]–[8].

From the simulation point of view, these devices pose the problem of properly defining the frequency signal to be used as input of the regulators. In fact, conventional electromechanical models for transient stability analysis structurally neglect frequency variations in transmission lines and loads. On the other hand, fully-fledged electromagnetical models, which would easily allow determining the frequency at any point of the system, are too computational demanding as the time scales of interest when dealing with frequency regulations are in the order of minutes, not milliseconds. This paper discusses the impact on power system transient stability analysis of different approaches to estimate the frequency at load buses, namely the Center of Inertia (COI), the *washout filter* and the *frequency divider formula*.

The evaluation of the COI is probably the most widely applied approach to estimate the frequency in a power system, due mainly to the simplicity of its computation, which is the weighted arithmetic mean of all synchronous machine rotor speeds of a power system. Several studies on frequency and transient stability analysis of power systems using the COI have been presented in the literature. Among these, we cite [9], [10].

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The second frequency estimation technique studied in this paper consists in computing the numerical derivative of the bus voltage phase angles through a washout filter [11]. Both analytical expressions and numerical methods have been proposed in the literature to define the numerical derivative of the voltage angle of a certain bus, e.g., in [12] and [13], respectively.

The last frequency estimator is a novel approach proposed by the authors in [14]. This novel estimator is based on the voltage divider concept, which results in a *frequency divider* formula based on synchronous machine rotor speeds and the network admittance matrix. Such a formula mimics the local behavior of the frequency at network buses by taking into account the electric distance to synchronous machines.

It is well-known that any control device is sensitive to its input signal. However, there is a lack of studies in the literature that compare the performance of frequency control devices depending on the frequency signal in power system simulators. This paper fills this gap, and provides a detailed comparison of the impact on transient stability analysis of the three frequency estimation approaches based on the COI, the washout filter, and the frequency divider formula proposed by the authors.

The remainder of the paper is organized as follows. In Section II three techniques to estimate the frequency in electromechanical power system models are presented. These are the frequency of the COI, a standard washout filter used in most common commercial software tools to estimate the frequency based on the bus voltage phase angles; and the frequency divider proposed in [14]. Section III presents two case studies based on the IEEE 14-bus system and the New England 39-bus, 10-machine systems. Finally Section IV duly draws conclusions.

II. Bus Frequency Estimation Approaches

This section presents the bus frequency estimation techniques considered in this paper, namely Center of Inertia (COI), Washout Filter (WF) and Frequency Divider (FD).

A. Center of Inertia

The most common technique to estimate the frequency of an interconnected ac transmission system is the COI, which is computed based on the rotor speeds and inertia constants of the synchronous generators connected to the system. Assuming a set $\mathcal{G}$ of synchronous generators, the expression to compute the COI is:

$$\omega_{\rm COI} = \frac{\sum_{j \in \mathcal{G}} H_j \omega_j}{\sum_{j \in \mathcal{G}} H_j} \tag{1}$$

where ω_j are rotor speeds and H_j are inertia constants.

The inertia-weighted nature of the COI makes this quantity particularly suited to study inter-area oscillations among machine clusters. However, local variations of the machines, especially those characterized by a small inertia, are lost. One can thus expect that the COI is not fully adequate to simulate local frequency controllers, as we duly discuss in the case study of this paper. Moreover, from the modeling point of view, it is unrealistic to assume that distributed generators, micrograds and consumers will receive the instantaneous signal of the COI frequency from the system operator. The frequency is actually very likely measured locally, using well-assessed techniques based on the sampling of ac quantities (see, for example, [15]). Thus, it is important to capture local variations of the frequency to properly model the response of such devices.

B. Washout Filter

The numerical derivative of the bus voltage phase angles is another well-known approach to estimate the frequency of ac transmission systems [16], [17]. As opposed to the COI, this technique can properly capture local oscillation modes but is prone to numerical issues. Figure 1 shows a typical implementation of the the numerical derivative, i.e., a washout filter and a low pass filter.

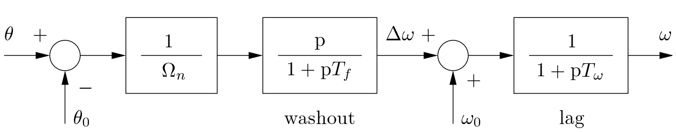

Fig. 1: Numerical derivative of the bus voltage phase angle composed of a washout and a low pass filters. θ_0 is the initial value of the bus voltage phase angle in radians and ω_0 is the synchronous frequency in pu.

The washout filter is necessary as the input quantity, i.e., the bus voltage phase angle θ, is an algebraic variable and thus can *jump* as a consequence of discrete events, such as faults and line outages. The discontinuity of the derivative of θ is the main issue of the WF approach. The low pass filter mitigates numerical issues but also introduces a delay that can be detrimental for the performance of local frequency controllers. A commonly accepted trade-off between accuracy and numerical efficiency is obtained with $T_f = 3/\Omega_n$ s and $T_\omega = 0.05$ s, where Ω_n is the nominal frequency of the system in rad/s. These are the values used in the simulations of this paper.

C. Frequency Divider

The COI and the WF are two well-accepted techniques to estimate the frequency of ac transmission systems. However, as discussed above, they show relevant technical, theoretical and/or numerical drawbacks. In [14], we have proposed an alternative approach, namely the FD formula, based on the augmented admittances matrix of the system and on the assumption that the frequency along the impedances of transmission lines varies as in a *continuum matter* where synchronous machine rotor speeds define boundary conditions.

A detailed discussion on the assumptions and hypotheses behind the definition of the FD formula are beyond the scope of this paper. Full details are provided in [14], to which we refer the interested reader. However, for the sake of completeness, in the remainder of this section, we briefly outline the expression and the features of the FD formula. This is as follows:

$$\boldsymbol{\omega}_B = \mathbf{1} + \mathbf{D}(\boldsymbol{\omega}_G - \mathbf{1}) \tag{2}$$

where $\boldsymbol{\omega}_G$ are machine rotor speeds, $\boldsymbol{\omega}_B$ are the frequencies at system buses, and $\mathbf{D}$ is defined as:

$$\mathbf{D} = -(\mathbf{B}_{BB} + \mathbf{B}_{G0})^{-1}\mathbf{B}_{BG} \tag{3}$$

where $\mathbf{B}_{BB}$ is the network susceptance matrix, i.e., the imaginary part of the standard network admittance matrix; $\mathbf{B}_{BG}$ is the susceptance matrices obtained using the internal impedances of the synchronous machines; and $\mathbf{B}_{G0}$ is a diagonal matrix that accounts for the internal susceptances of the synchronous machines at generator buses.

The FD formula (2), while approximated, solves the main issues of the COI and washout filter. This formula, in fact, is able to capture local oscillations as the rotor speeds are weighted based on their electrical proximity, not their inertia. The frequencies $\boldsymbol{\omega}_B$ calculated with (2) are also free from numerical inconsistencies as (2) is a linear expression of machine rotor speeds, which are, by definition, continuous and smooth state variables.

III. Case Study

This section compares the performance of frequency control devices in a power system when their input signal is provided by the WF and the FD, as well as when the signal is the frequency of the COI. Thermostatically Controlled Loads (TCLs) are considered for the comparison, and their model is described in Appendix A. With this aim, two benchmark networks are used in this study: the IEEE 14-bus system (Subsection III-A), and the New England 39-bus, 10-machine system (Subsection III-B).

All simulations are obtained using Dome, a Python-based power system software tool [18]. The Dome version utilized in this case study is based on Python 3.4.0; ATLAS 3.10.1 for dense vector and matrix operations; CVXOPT 1.1.8 for sparse matrix operations; and KLU 1.3.2 for sparse matrix factorization. All simulations were executed on a 64-bit Linux Ubuntu 14.04 operating system running on a 8 core 3.60 GHz Intel Xeon with 12 GB of RAM.

A. IEEE 14-bus test system

This subsection considers the IEEE 14-bus test system (see Fig. 2). This benchmark network consists of 2 synchronous machines and 3 synchronous compensators, 2 two-winding and

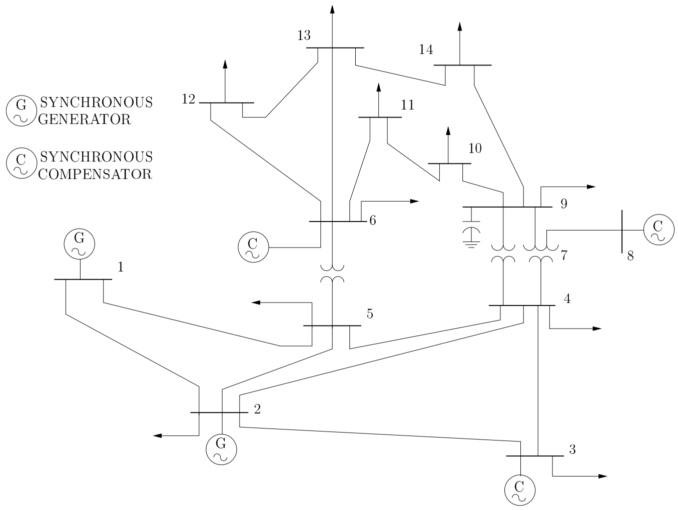

Fig. 2: IEEE 14-bus test system.

1 three-winding transformers, 15 transmission lines and 11 loads. The system also includes primary voltage regulators (AVRs). All dynamic data of the IEEE 14-bus system as well as a detailed discussion of its transient behavior can be found in [19]. For this scenario, primary and secondary frequency regulation are not included in order to study the effect of the frequency regulation of the TCLs solely. The contingency is the outage of the line connecting buses 2 and 4 in base loading conditions, as well as with 20% of overload. The amount of TCLs is 30% of the total load.

Base Loading Conditions: Figure 3 shows the rotor speed of the synchronous machine at bus 2, the frequency of the bus estimated by the WF and the FD, and the frequency of the COI, for the base case loading conditions and without TCLs. While both WF and FD estimators show a trend similar to the rotor speed of the machine, the signal provided by the FD appears to be more accurate, since it includes oscillations of same period and similar amplitude than those of the rotor speed. On the other hand, the frequency of the COI "filters" such oscillations, providing only information on the average frequency variation.

In this case, the differences in the estimated frequencies appear to be negligible when including the TCLs, as shown in Fig. 4. The rotor speed of the machine in bus 2 is depicted for the cases without and with TCLs. In the latter case, we compare the dynamic response of TCLs in three scenarios, corresponding to using as control input signals the frequency estimation provided by the WF, the COI and the FD, respectively. The inclusion of the TCLs reduces the frequency drop by about 50% for the three cases.

While in this case the transient response of the system does not appear to be affected by the model of the frequency estimation, one cannot conclude that this is always the case. In the next subsection, in fact, we show that, depending on the loading level, the choice of the input signal of the TCLs can provide a considerably different performance of these devices, and therefore, a different behavior of the whole system.

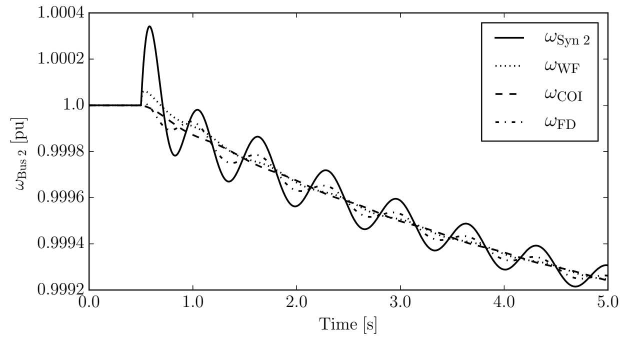

Fig. 3: Frequency of bus 2 when TCLs are not included.

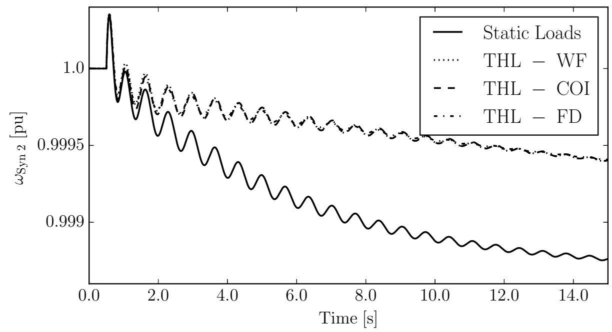

Fig. 4: Rotor speed of the synchronous generator in bus 2 when TCLs are included.

20% Load Increase: Figure 5 is obtained for a 20% increase of the load with respect to the base case without TCLs and shows the trajectories of the rotor speed of the machine at bus 2 with the signals given by the WF, the COI and the FD. Undamped oscillations can be observed due to the presence of a limit cycle (Fig. 5(a)). From Fig. 5(b), it can be seen that the FD is again the most accurate, while the WF includes a delay in the signal, and the COI contains counter-phase oscillations due to the large size of the synchronous machine at bus 1.

Depending of the chosen frequency estimation technique, considerably different input signals are introduced into the TCLs controllers. This is shown in Fig. 6, where the estimated frequencies at the load bus 14 are compared.

Finally, the rotor speed of the machine at bus 2 is shown in Fig. 7, which is obtained including TCLs. The frequency drop is again reduced by about 50%. However, while the WF and the COI signals lead to a stationary limit cycle, the usage of the FD indicates that oscillations are actually damped. It is worth noticing that the only difference in the model is the formula to estimate the frequency signal sent to the TCL. Hence, we conclude that the choice of the techniques to estimate the frequency can affect considerably the dynamic response of the system, especially if devices controlling the frequency, such as TCLs, are considered. In this case, the WF and the COI appear to be more conservative from the control point of view. This

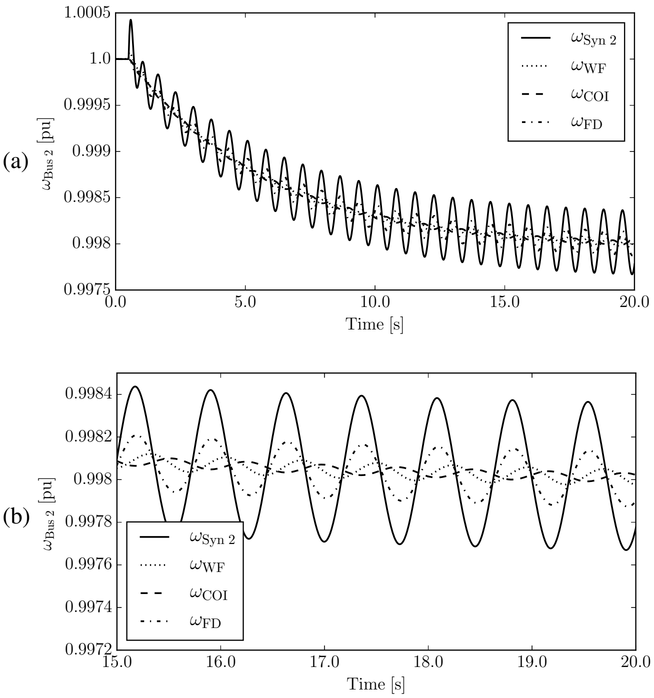

Fig. 5: Frequency of bus 2 with 20% of system overload and when TCLs are not included.

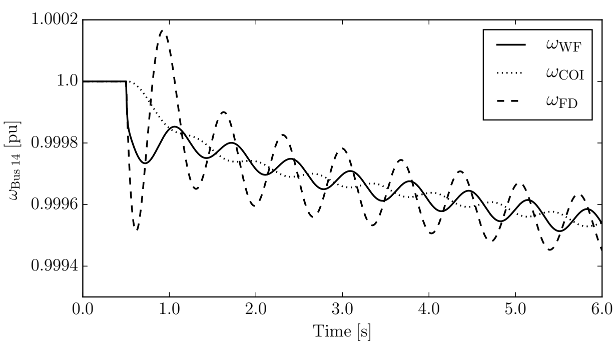

Fig. 6: Control input signals of the TCLs at bus 14 using WF, FD and the COI.

is a consequence of the fact that these models do not capture the variations of local bus frequencies as accurately as the FD.

B. New England 39-bus, 10-machine test system

The single-line diagram of the New England 39-bus, 10-machine test system is depicted in Fig. 8. This benchmark network contains 19 loads totaling $7,316.5$ MW and $1,690.9$ MVAr of active and reactive power, respectively (20% load increase with respect to the base case is assumed). The system model also includes generator controllers such as primary voltage regulators, as well as both primary and secondary frequency regulation (turbine governors and AGC). Dynamic data of the New England 39-bus, 10-machine system can be

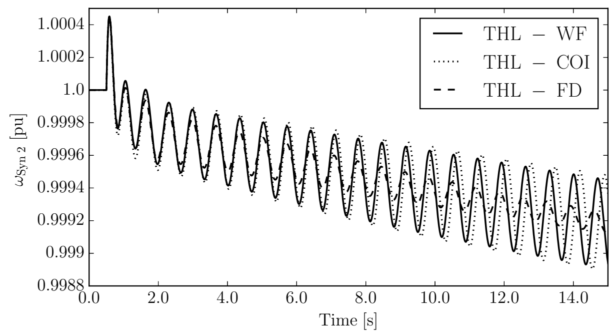

Fig. 7: Rotor speed of the synchronous generator in bus 2 with 20% of system overload and TCLs.

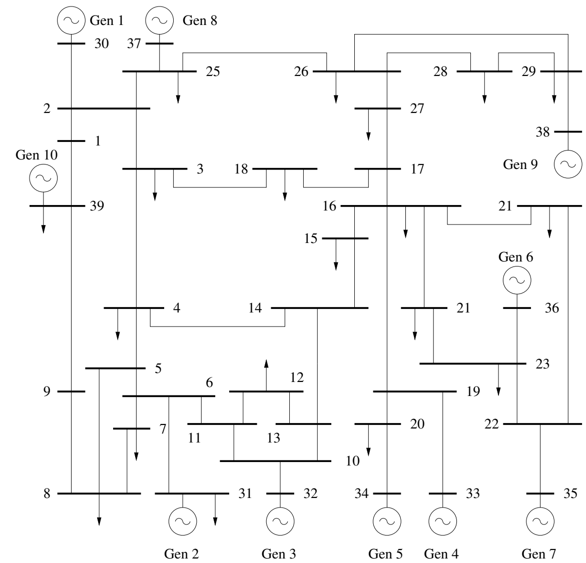

Fig. 8: New England 39-bus, 10-machine system.

found in [20]. The contingency is a three-phase fault at bus 21, cleared by the opening of the line connecting buses 16 and 21 after 160 ms. TCLs are the 20% of the total load.

Figure 9 shows the rotor speed of the machine at bus 31 (Gen 2) without TCLs, and with TCLs considering the three control input signals, namely FD, WF and COI. Three different models of the synchronous machines of the system are compared: the one-axis 3^{rd} order model (Fig. 9(a)), the Sauer and Pai's 6^{th} order model (Fig. 9(b)), and the fully-fledged 8^{th} order model (Fig. 9(c)) [19].

Figure 9(a) shows that including TCLs into the system allows reducing the frequency variations due to the fault. The TCLs reduces the damping of dominant modes. Such modes appear to be slightly better damped if the model includes the FD signal.

Figures 9(b) and 9(c) show the response of the system for more detailed and accurate models of the synchronous machines. In these cases, the dynamic interaction of the machine transient and subtransient dynamics with the TCLs causes poorly damped frequency oscillations. It is interesting to note that, when considering stator flux dynamics and the WF signal (see Fig. 9(c)) such oscillations become unstable

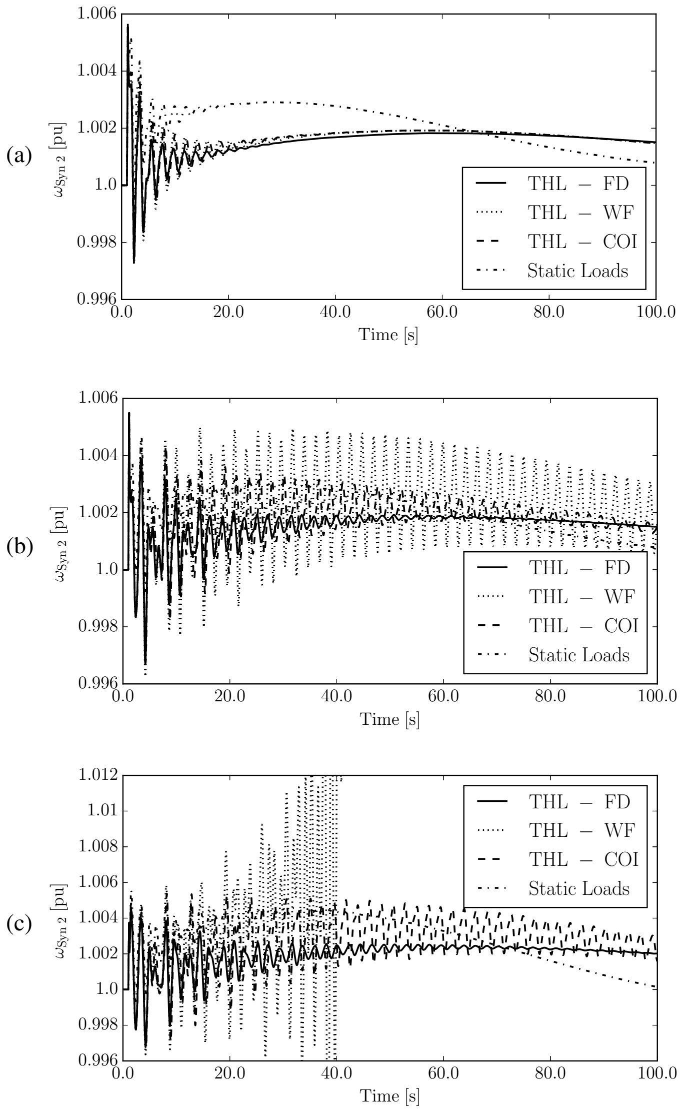

Fig. 9: Rotor speed of the synchronous generator in bus 31 (Gen 2). (a) 3^{rd} order synchronous machine model. (b) 6^{th} order synchronous machine model. (c) 8^{th} order synchronous machine model.

and lead the system to collapse. On the other hand, the TCLs controller coupled to the FD shows that oscillations are properly damped.

IV. Concluding Remarks

This paper compares different techniques to estimate the frequency for electromechanical models of power systems. These are the frequency of the center of inertia, the commonly-used washout filter of bus voltage phase angles, and a voltage divider-based formula, proposed by the authors in [14].

The following remarks are based on the simulation results presented in the paper.

i. The numerical derivative of the bus voltage phase angle of the WF can lead to non-physical oscillations and, possibly to numerical instabilities.
ii. The average rotor speed provided by the COI filters local frequency variations. This fact may cause poorly-damped frequency oscillations, especially if coupled to devices with a slow response, such as TCLs.
iii. Controllers using signals obtained with the frequency divider are less prone to introduce undamped oscillations.

From the results above, it is clear that a proper modeling of the control signals can make a significant difference in the transient stability analysis of a power system with inclusion of frequency controllers other than primary frequency regulators of synchronous machines. We believe that the definition of a criterion to estimate the fidelity of such power system models is an interesting and urgent research topic.

Appendix A
Thermostatically Controlled Loads

TCLs are dynamic loads with temperature control [21]. These can be air conditioning systems, industrial refrigerators or heating systems. In most cases, the reference temperature is fixed to an assigned value. There are, however, prototypes of TCLs that include a measure of the system frequency and that vary the reference temperature in order to reduce frequency deviations [6], [22], [23].

The control scheme of the TCL is depicted in Fig. 10. The meaning of the variables are the following: Θ is the load temperature (lumped model); Θ_a is the ambient temperature; g is the equivalent load conductance, v is the load terminal voltage; and P the consumed active power.

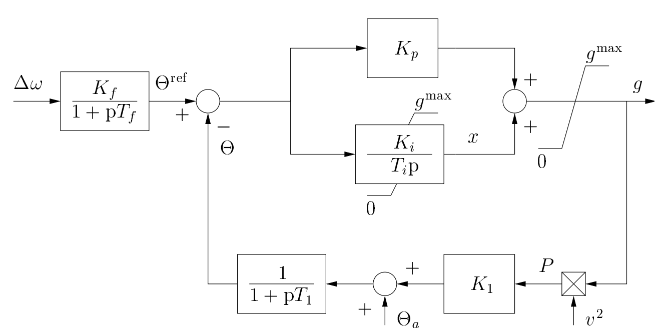

Fig. 10: Thermostatically controlled load with frequency control.

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