A Short-Term Dynamic Electricity Market Model with Memory Effect

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***Abstract*—This paper presents a short-term dynamic electricity market model with "memory effect." The model takes into account how the memory of market participants, in this work the memory of suppliers, impacts on their behavior, i.e., on their bids. The memory effect is represented by employing proper mathematical tools from the theory of fractional calculus. The impact of the proposed fractional-order model on system dynamics and, in particular, its interaction with a secondary frequency control, are studied by means of the well-known WSCC 9-bus system. The proposed model is also compared with a conventional, integer-order electricity market model. Results indicate that the inclusion of memory leads participants, e.g. suppliers, to adopt a conservative behavior.**

***Index Terms*—Dynamic electricity market, memory effects, fractional calculus, power systems.**

I. Introduction

Short-term electricity markets, e.g. balancing markets, are becoming increasingly important as the integration of variable renewable energy sources like wind and solar continues [1]. These markets offer flexibility to the power system by maintaining the power balance until physical generation and consumption. The timescale of these markets is comparable with those of the long-term power system dynamics, i.e. secondary frequency control [2]. Because of the similar timescales, there is a concern on the coupling between the dynamic response of the system and such markets [3]. The first studies that have looked at this problem date back to around two decades ago when Alvarado used first-order differential equations to describe the dynamics of a short-term electricity market model [4]. This model is an abstraction but allows studying the stability of the markets as well as interactions with the power grid [5]. Numerous works have used this model to date. For example, we cite [6] and [7].

In this work, we propose an approach to include in the dynamic model [4] a specific aspect of the behavior of the market participants such as suppliers, namely the "memory effect." This aspect is absent in the several variants of the Alvarado's model proposed so far in the literature. These models, in fact, are all based on conventional integer-order time derivatives. These consider an infinitely small neighborhood of the "present" time, i.e. the time at which the derivatives are computed [8]. This means that, by definition, the processes that are modeled through conventional differential equations have infinitely fast "amnesia." However, taking into account the memory of market participants is of utmost importance in economic processes as they can remember the changes of economic indicators and factors in the past [9]. These changes can then impact their behavior and decisions.

An effective and powerful tool to model the memory effects of a dynamic system is fractional calculus, which deals with the analysis of non-integer order differentials and integrals [10]. The ability of fractional derivatives to capture physical processes better than traditional integer-order derivatives has been shown for a number of systems in physics and engineering [11]. For example, reference [12] provides a review of the application of fractional calculus in science and engineering. The recent work in [13] proposes a new economic model of the price dynamics of goods that takes into account the memory of the market agents. Using the work in [13] as a source of inspiration, this paper takes into account for the first time the memory of the participants, in particular, suppliers, in power system markets.

The specific contributions of the paper are as follows:

- A dynamic electricity market model with inclusion of memory effect. The memory is represented through fractional-order derivatives.
- An in-depth comparison of the impact of fractional and integer-order market models on the decision-making process of suppliers, as well as on the overall dynamic performance of the grid.

The remainder of the paper is organized as follows. Section II recalls Alvarado's dynamic market model. Section III describes the modeling of economic processes with memory using fractional calculus. Section IV presents the proposed fractional-order market model. Section V compares the impact of integer and fractional market models on the decision-making of suppliers and on power system dynamics. Finally, conclusions and future work are discussed in Section VI.

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II. Dynamic Electricity Market Model

Reference [5] proposes a dynamic market model to study the couplings between the dynamics of the power network and the short-term electricity market. The model is formulated as:

$$T_\lambda \frac{d\lambda(t)}{dt} = K_E(\omega^{\mathrm{ref}} - \omega_{\mathrm{CoI}}(t)) - H_d\lambda(t)\,, \tag{1}$$

$$T_{gi}\frac{d\Delta p_{gi}(t)}{dt} = \lambda(t) - c_{gi}\Delta p_{gi}(t) - b_{gi}\,, \quad i = 1, \dots, n_g\ , \tag{2}$$

where $\lambda(t)$ is the electricity price; ω^{ref} represents the reference frequency; $\omega_{\mathrm{CoI}}(t)$ represents the frequency of the Center-of-Inertia (CoI); $\Delta p_{gi}(t)$ is the variation of the i-th generator active power; c_{gi} and b_{gi} are the parameters of the marginal cost of the i-th generator; T_λ, T_{gi} are time constants; H_d is the deviation with respect to a perfect tracking integrator; and K_E is the feedback gain. Equation (1) accounts for the system power imbalance indirectly, i.e., through the deviation frequency of the CoI with respect to the reference frequency, i.e. $\omega^{\mathrm{ref}} - \omega_{\mathrm{CoI}}(t)$. Finally, equation (2) assume that a generator will increase/decrease its power production if the electricity price $\lambda(t)$ is higher/lower than its marginal cost. Note that, in the original model proposed in [5], also the loads are included in the market using an expression similar to (2). In the remainder of this paper, however, we assume that loads are inelastic.

The market model (1)-(2) has a very similar structure to that of a conventional secondary frequency control, i.e. the automatic generation control (AGC) [14]. To better illustrate similarities, the control diagrams of a conventional AGC and that of the market model (1)-(2) (or MAGC) are depicted in Fig. 1 and Fig. 2, respectively. It can be seen that the input of both controllers is the same. The AGC includes an integrator with gain K_o that has a similar function with the Low-Pass Filter block of the market, namely, to reduce the frequency oscillations. Finally, the outputs of the AGC and MAGC are distributed to the turbine governors (TGs) of the synchronous generators proportionally to their droops (R_i) and bids, respectively.

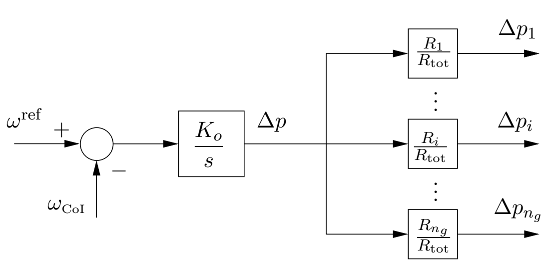

Fig. 1: AGC control diagram.

III. Modeling Economic Processes with Memory

A mathematical tool that allows modeling the memory effects of market agents is fractional-order differential equations. The recent work [13] extends the well-known Evans model – that describes the price dynamics of goods [15] – to take into account the memory of market participants. Based on this theoretical background, this section shows the mathematical steps that lead to the proposed fractional market model.

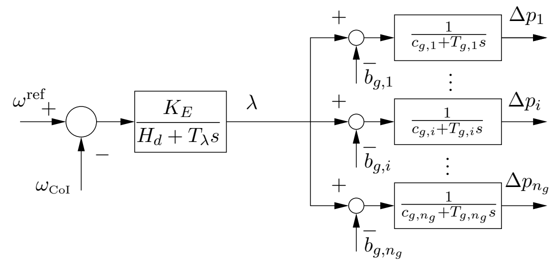

Fig. 2: MAGC control diagram.

The Evans model of price dynamics is as follows:

$$\frac{d\Lambda(t)}{dt} = -\gamma \frac{dQ(t)}{dt}\,, \tag{3}$$

where $\Lambda(t)$ represents the price of goods; γ is a proportional coefficient that represents the speed of response; and $Q(t)$ represents the stocks. The change in stocks is defined by the following differential equation:

$$\frac{dQ(t)}{dt} = S(t) - D(t)\,, \tag{4}$$

where $S(t)$ and $D(t)$ represent the supply and demand for goods, respectively. Using equation (4), equation (3) can be rewritten as:

$$\frac{d\Lambda(t)}{dt} = -\gamma(S(t) - D(t))\,. \tag{5}$$

The supply and demand are described by the following equations:

$$\begin{aligned} S(t) &= c_s + b_s\Lambda(t)\,, \\ D(t) &= d_d + a_d\Lambda(t)\,, \end{aligned} \tag{6}$$

where a_d, b_s, c_s, d_d are constant parameters. In particular, c_s and d_d represent the supply and demand, respectively, and do not depend on the price $\Lambda(t)$. In general, it is assumed that $a_d < 0$ and $b_s > 0$. Using equation (6), equation (5) of price dynamics can be rewritten, as follows:

$$\frac{d\Lambda(t)}{dt} + \gamma(b_s - a_d)\Lambda(t) = \gamma(d_d - c_s)\,. \tag{7}$$

Equation (7) is a first-order differential equation. It cannot account for the memory of the market participants. With this aim, one has to consider not only the difference $S(t) - D(t)$, but also the "history" of changes of the differences $S(\tau) - D(\tau)$ on a finite time interval $\tau \in [0, t]$. Such a dependence of $\Lambda(t)$ can be described by the following equation:

$$\begin{aligned} \frac{d\Lambda(t)}{dt} &= -\int_0^t \gamma(t-\tau)\frac{dQ(\tau)}{d\tau}d\tau \\ &= -\int_0^t \gamma(t-\tau)(S(\tau) - D(\tau))d\tau\,, \end{aligned} \tag{8}$$

where $\gamma(t)$ represents the memory function. Assuming a power-law fading memory, the function $\gamma(t)$ can be written as follows:

$$\gamma(t-\tau) = \frac{\gamma}{\Gamma(\alpha)}(t-\tau)^{\alpha-1}\,, \tag{9}$$

where $\Gamma(\alpha)$ is the gamma function; $0 < \alpha < 1$ is the fractional-order; and $t > \tau$. Using (9), (8) can be rewritten as:

$$\begin{aligned}\frac{d\Lambda(t)}{dt} &= -\gamma\left(I^{\alpha}_{RL;0+}Q^{(1)}\right)(t)\\ &= -\gamma\left(I^{\alpha}_{RL;0+}(S-D)\right)(t)\,,\end{aligned} \tag{10}$$

where $I_{RL;0+}$ is the Riemann-Liouville fractional integral defined as:

$$I^{\alpha}_{RL;0+}f(t) = \frac{1}{\Gamma(\alpha)}\int_0^t (t-\tau)^{\alpha-1}f(\tau)d\tau\,, \tag{11}$$

where the function $f(\tau)$ is measurable on the interval $(0,t)$ and has the property $\int_0^t |f(\tau)|d\tau < \infty$. Since we are interested to obtain a differential equation for the price dynamics, we act on equation (10) by the left-sided Caputo derivative of order $\alpha > 0$ defined as follows:

$$\frac{d^{\alpha}}{dt^{\alpha}}_{C;0+}f(t) = \frac{1}{\Gamma(n-\alpha)}\int_0^t (t-\tau)^{n-\alpha-1}f^{(n)}(\tau)d\tau\,, \tag{12}$$

and obtain the following fractional differential equation:

$$\frac{d^{\alpha+1}}{dt^{\alpha+1}}_{C;0+}\Lambda(t) = -\gamma(S(t)-D(t))\,, \tag{13}$$

where $n-1 < \alpha+1 \leq 1$ and $n \in \mathbb{N}$. It can be seen that for $\alpha = 0$, equation (13) takes the form of the equation (7), while for $\alpha = 1$, equation (13) takes the form of the second-order differential equation of the Evans model [13].

IV. Dynamic Electricity Market Model with Memory Effect

The short-term electricity market model (1)-(2) cannot capture the memory of market participants as it uses first-order integer derivatives. On the other hand, the previous section shows that the memory of market participants can be taken into account using the mathematical tool of fractional calculus. Motivated by this theoretical background, we propose the following fractional-order version of the dynamic electricity market model (1)-(2):

$$T_{\lambda}\frac{d\lambda(t)}{dt} = K_E(\omega^{\mathrm{ref}} - \omega_{\mathrm{CoI}}(t)) - \lambda(t)\,, \tag{14}$$

$$T_{gi}\frac{d^{\alpha}\Delta p_{gi}(t)}{dt^{\alpha}} = \lambda(t) - c_{gi}\Delta p_{gi}(t) - b_{gi}\,, \quad i = 1,\dots,n_g\,, \tag{15}$$

where $0 < \alpha < 1$ is the fractional order. The new market model (14)-(15) accounts for the memory of generators through the fractional equation (15) of the generator dynamics. Note that in this work we consider a conventional power system with synchronous generators. Non-dispatchable generation, such as wind and solar energy resources, if bidding in the electricity market, can be also modeled using (15). A case study that considers wind generation and the market model (1)-(2) with $\alpha = 1$ can be found in [14].

A. Oustaloup's Recursive Approximation

In order to implement or simulate in practice the proposed fractional market model (14)-(15), one needs to approximate the fractional dynamics, in this case equation (15), with rational order transfer functions [16]. In this work, we select the Oustaloup's Recursive Approximation (ORA) method to approximate the fractional generator dynamics. The generalized ORA of a fractional derivative of order α is defined as [17]:

$$s^{\alpha} \approx \omega_h^{\alpha}\prod_{k=1}^{N}\frac{s+\omega'_k}{s+\omega_k}\,, \tag{16}$$

where $\omega'_k = \omega_b\omega_v^{(2k-1-\alpha)/N}$, $\omega_k = \omega_b\omega_v^{(2k-1+\alpha)/N}$, $\omega_v = \sqrt{\omega_h/\omega_b}$. In the above expressions, $[\omega_b, \omega_h]$ is the frequency range for which the approximation is designed to be valid; N is the order of the polynomial approximation. The term "generalized" means that, in (16), N can be either even or odd [17], while the term "recursive" implies that the values of ω'_k, ω_k result from a set of recursive equations [18]. The block diagram of ORA is shown in Fig. 3. Further details on the ORA method and its accuracy can be found in [16] and references therein.

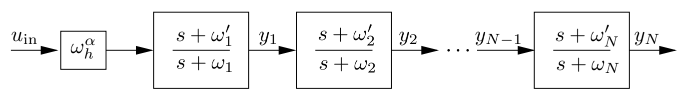

Fig. 3: Oustaloup's recursive approximation block diagram.

V. Case Study

In this section, we study the dynamic behavior of the two dynamic market models introduced in Sections II and IV, namely, conventional integer-order MAGC (I-MAGC) (1)-(2) and fractional-order MAGC (F-MAGC) (14)-(15). The objective is to evaluate the impact of these models on the behavior of market participants, e.g. generator schedules, and on the overall dynamic response of the power system. With this aim, we first discuss the impact of I-MAGC and illustrate the dynamic coupling between market and AGC in Section V-A. Section V-B discusses a sensitivity analysis with respect to the fractional-order α of the F-MAGC and compares results with the I-MAGC. Both Sections V-A and V-B consider a 10% sudden load increase. The dynamic response of both I-MAGC and F-MAGC following a 10% sudden load decrease is presented in Section V-C.

The case study is based on a modified version of the well-known WSCC 9-bus test system, whose details are provided in [14]. All simulations are performed using the power system analysis software tool Dome [19].

A. Impact of the Frequency of Price Updates

Some long-term power system dynamics, e.g. the dynamics of the AGC, evolve with a timescale similar to today's short-term market dynamics [2]. For this reason, it is important to understand how the frequency with which the market price is updated impacts on the decision-making process of market participants and on power system dynamics. In the continuous market models considered in this paper, the information on how often the price is updated is contained in the value of the gain K_E in (1). Hence, this section presents a sensitivity analysis with respect to the variations of the K_E.

Figure 4 shows that the value of K_E has a negligible impact on the overall dynamic of the system, i.e. the frequency nadir is the same in all cases. This was expected as the MAGC is slow with respect to the primary frequency control. Figure 5, on the other hand, shows that the schedule of generator active power are by the value of K_E. Specifically, the faster the price updates, i.e. the higher K_E, the faster the generator response and consequently the higher the generator schedules. This phenomenon is called *price chasing* [20]. These results indicate that how often the market updates the price (which in this continuous model is modeled by means of K_E) impacts the schedule of the suppliers or generators.

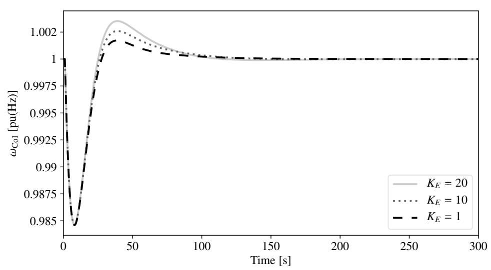

Fig. 4: Trajectories of the frequency of the CoI.

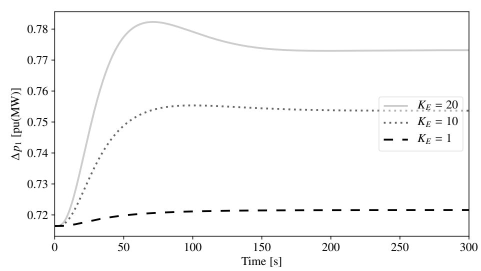

Fig. 5: Trajectories of the MAGC active power schedules of generator 1.

The trajectories of the AGC set-point Δp_1 of generator 1 are shown in Fig. 6. Higher gain values – and hence faster price updates – lead to faster AGC response and lower AGC set-points. This has to be expected as the AGC has to compensate the difference in the market schedules since at the end the total power output of the generator has to be the same. These results imply that, depending on the market design and rewards of the ancillary services, generators may prefer to compensate power imbalances through the short-term market or through the secondary frequency control.

B. Sensitivity Analysis

This section presents a sensitivity analysis with respect to the fractional-order α of the F-MAGC and compares the results with the ones obtained using the conventional integer-order market model. For fair comparison, all other parameters,

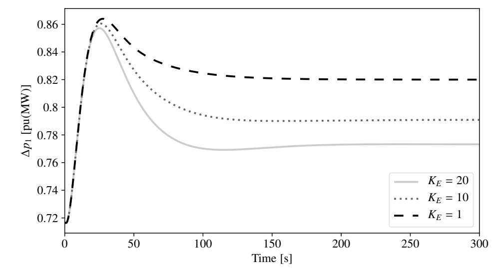

Fig. 6: Trajectories of the AGC active power set-point of generator 1.

namely time constants and gains, of both market models, (1)-(2) and (14)-(15), are kept the same.

First we compare the impact of I-MAGC and F-MAGC on power system dynamics. Figure 7 shows the trajectories of $\omega_{\rm CoI}$ for both models. It is interesting to observe that both the I-MAGC and F-MAGC lead to the same frequency nadir and very similar frequency overshoots. The memory of market participants, thus, does not have a relevant impact on the overall power system dynamics. These results are consistent with those shown in Fig. 4.

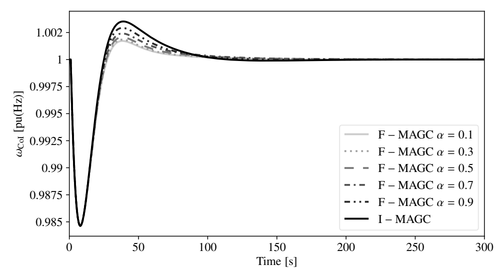

Fig. 7: Comparison of the trajectories of the frequency of the CoI as obtained with the I-MAGC and F-MAGC.

Next we compare the impact that different values of α have on the behavior of the generators. Figure 8 shows that the F-MAGC leads to different (in this case, lower) market schedules as compared to that of the I-MAGC. This result suggests that the F-MAGC is less prone to the price changes. In other words, taking into account the memory of market participants makes them more conservative. Furthermore, the higher the fractional-order α, the faster the generator response, and consequently the higher the generator market schedules. This conclusion is supported by Fig. 9. This figure shows that the AGC set-point for the fractional market is less prone to changes compared to the conventional market. Observe that, in steady-state, the total active power generation depends only on the variation of the load consumption. However, α changes the quota of active power produced by each machine. The dependency of the steady-state on α is evident in Fig. 8. However, for $\alpha > 0.9$ and $\alpha \to 1$, the steady-state operating point of each machine varies very little.

C. Impact of a Sudden Load Decrease

In this final example, we compare the impact on the performance of I-MAGC and F-MAGC of a 10% sudden load decrease occurring at $t = 1$ s.

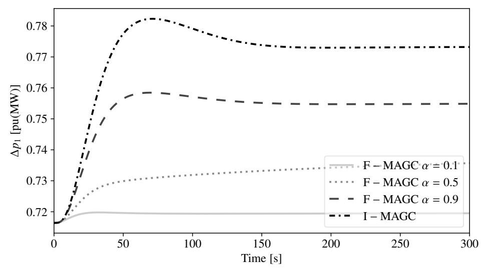

Fig. 8: Trajectories of the MAGC active power schedules of generator 1.

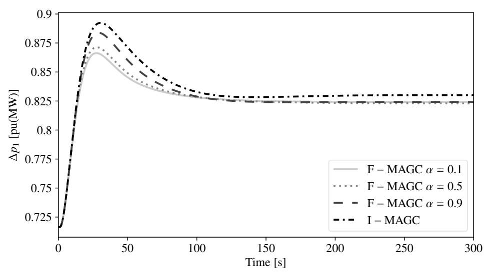

Fig. 9: Trajectories of the AGC active power set-point of generator 1.

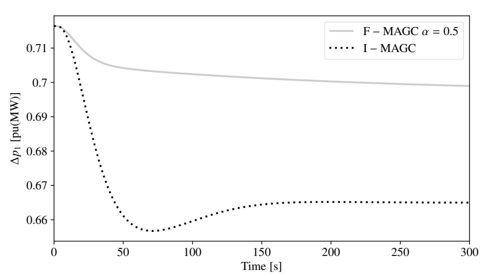

Fig. 10: Trajectories of the MAGC active power schedules of generator 1.

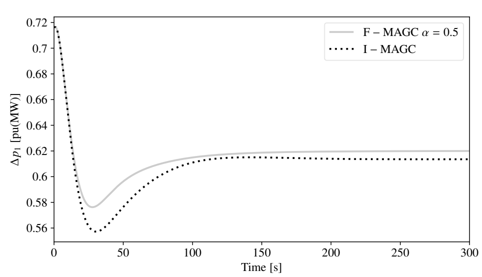

Fig. 11: Trajectories of the AGC active power set-point of generator 1.

Figure 10 shows that the F-MAGC is again less prone to price changes compared to the I-MAGC. For the considered contingency, such a behavior leads the market to schedule higher generator powers.

Figure 11 shows the AGC power output and indicates that the I-MAGC case responds faster than the F-MAGC to the contingency. This result is consistent with that obtained in the previous section, i.e. the memory effect makes the generators less sensitive to changes in the operating point of the grid. This conservativeness, however, has to be compensated, at least in the short term, by the secondary frequency regulation.

VI. Conclusions

This paper proposes a fractional dynamic electricity market model. The model takes into account the memory of suppliers through fractional calculus. Results indicate that the memory effect leads to a conservative behavior of suppliers and their decisions. This paper is a very first attempt to model the behavior of market participants and its impact on their decisions and on power system dynamics. We believe that this work poses the basis for interesting future developments.

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