PMU-based Estimation of the Frequency of the Center of Inertia and Generator Rotor Speeds

Muyang Liu, *Student Member, IEEE*, Álvaro Ortega, *Member, IEEE*, Federico Milano, *Fellow, IEEE*
School of Electrical and Electronic Engineering, University College Dublin, Ireland
muyang.liu@ucdconnect.ie, {alvaro.ortegamanjavacas, federico.milano}@ucd.ie

***Abstract*—The paper compares a variety of on-line approaches based on PMUs to estimate the angular speed of individual synchronous machines as well as of the center of inertia. These approaches involve the solution of an optimization problem or a Weighted Least Square problem and are based on the frequency divider formula that has been recently proposed on the Transactions on Power Systems by the second and third authors. The case study is based on a dynamic 1,479-bus model of the all-island Irish system with inclusion of stochastic wind speed, noise and time-varying PMU measurement delays. The scenarios studied in the paper allow identifying the key features of the considered estimation approaches. A thorough discussion on the impact of measurement delays and system size is also provided.**

***Index Terms*—Frequency estimation, frequency divider, phasor measurement unit, measurement delay, synchronous machine, center of inertia.**

I. Introduction

A. Motivation

On-line estimations of synchronous machine rotor speeds and of the frequency of the Center of Inertia (COI) are meaningful to Transmission System Operators (TSOs). These signals can be used to solve preventive transient stability analysis or utilized as input signals to centralized [1], [2] or decentralized controllers [3]. While there exist attempts to dynamically estimate the rotor speeds of synchronous machines [4], [5], the calculation of the COI has been confined so far to simulations [6], [7]. TSOs generally use the frequency estimation of a pilot bus instead. Feasible techniques to estimate both rotor speeds and the frequency of the COI have been recently developed by the second and third authors [8]–[10]. This paper compares the accuracy and robustness of conventional estimation approaches with the novel alternatives discussed in [9] and [10] through several real-world scenarios.

B. Literature Review

In recent years, Phasor Measurement Units (PMUs) have become a common device to measure current and voltage phasors, as well as the frequency of these quantities at the buses where they are installed [11]. Measuring local bus frequencies allow PMUs to be utilized to effectively set up an on-line dynamic state estimation. Relevant works in this area are [12]–[16]. However, PMU-based estimations are prone to accuracy issues due to latency, bad data and noise [17].

Among the aforementioned issues, latency is one of the most significant ones for on-line dynamic state estimation. Sampling, concentration and propagation of measurement data inevitably introduce latency [18]. Then, data packet loss and disorder during the communication from PMUs to the control center introduce randomness and lead to quasi-periodic delays [19].

PMU measurement delays are generally modelled as constant or purely stochastic [20]–[22]. These models, however, are inaccurate as they cannot capture the quenching phenomenon, which often occurs in dynamic systems with periodic time-varying delays. The quenching phenomenon consists in an erroneous estimation of the stability of a delayed system. This is why, more often than not, constant and purely stochastic delay models lead to conclude that a system is unstable when it is not or *vice versa* [23].

Reference [24] proposes a realistic delay model for PMU measurements. Such a model consists of three components: constant, stochastic and pseudo-periodic components. This composite model, to the knowledge of the authors, is currently the most precise model to mimic the realistic PMU measurement delay.

C. Contributions

The contributions of the paper are the following.

- A comprehensive study of the accuracy of the on-line estimation approaches proposed in [9], [10] under the combined impact of dynamic wind speed, noise and realistic measurement delays.
- A comparison of centralized and decentralized estimation approaches of the rotor speed of synchronous machines considering their different communication latencies in real-world applications.
- A comparison of different COI frequency estimation approaches based on (i) the formula provided in [9]; and (ii) only one measure of a relevant (pilot) bus.

All comparisons are based on a real-world power system model, namely the all-island Irish system, which allows properly discussing the impact of topology and system size on the accuracy of the frequency estimation.

This work was supported by: Science Foundation Ireland, by funding Muyang Liu and Federico Milano under Grant No. SFI/15/IA/3074; and the European Union's Horizon 2020 research and innovation programme, by funding Álvaro Ortega and Federico Milano under Grant No. 727481.

D. Organization

The paper is organised as follows. Section II briefly reviews the PMU-based on-line estimation approaches of the rotor speed of synchronous machines and the frequency of the COI. Section III provides the case study based on a 1,479-bus dynamic model of the all-island Irish transmission system (AIITS) with bus noise, Weibull distributed wind speed, and PMU measurement delay, and considering measure failure scenarios. Section IV provides conclusions and outlines future work.

II. Estimation Theory

This section briefly outlines the theory of the proposed on-line estimation approaches of synchronous machine rotor speeds and the frequency of the COI. Subsection II-A introduces both the novel optimization and the conventional Weighted Least Square (WLS) method to estimate the generator rotor speeds, and outlines their comparisons discussed in [10]. Subsection II-B reviews the rotor speed-free estimation of the frequency of the COI discussed in [9].

A. Synchronous Machine Rotor Speed Estimation

1) Optimization Problem: The proposed synchronous machine rotor speed estimation is formulated as the following optimization problem:

$$\begin{aligned} \min_{(\boldsymbol{e}_{\mathrm{B}},\, \Delta\boldsymbol{\omega}_{\mathrm{G}})} \quad & J = \frac{1}{2}\boldsymbol{e}_{\mathrm{B}}^{T}\mathbf{W}\boldsymbol{e}_{\mathrm{B}} \\ \text{s.t.} \quad & \mathbf{0} = \mathbf{B}_{\mathrm{BG}}\Delta\boldsymbol{\omega}_{\mathrm{G}} + \mathbf{B}_{\mathrm{BB}}(\Delta\tilde{\boldsymbol{\omega}}_{\mathrm{B}} + \boldsymbol{e}_{\mathrm{B}}) \quad : \boldsymbol{\mu}_{\mathrm{B}} \ , \end{aligned} \tag{1}$$

where $\Delta\tilde{\boldsymbol{\omega}}_{\mathrm{B}}$ is the input vector of measured bus frequency deviations as provided by the PMUs; $\boldsymbol{e}_{\mathrm{B}}$ is the vector of measurement errors; $\Delta\boldsymbol{\omega}_{\mathrm{G}}$ is the vector of estimated rotor speed deviations of the synchronous machines; $\boldsymbol{\mu}_{\mathrm{B}}$ are the dual variables associated with the equality constraints; and $\mathbf{W}$ is the weight matrix defined by the variance of measurement errors. Reference [10] deduces the first order optimality conditions of the Lagrangian function $\mathcal{L}(\boldsymbol{\omega}_{\mathrm{G}}, \boldsymbol{e}_{\mathrm{B}}, \boldsymbol{\mu}_{\mathrm{B}})$ of (1), which can be used to find the optimal solution and compute the sensitivities of each variables.

2) Weighted Least Square Problem: Reference [10] also provides an optimal formula of the conventional linear measurement problem of the synchronous machine rotor speed based on the Frequency Divider Formula (FDF):

$$\Delta\boldsymbol{\omega}^{*}_{\mathrm{G}} = (\mathbf{D}^{T}\mathbf{D})^{-1}\mathbf{D}^{T}\Delta\tilde{\boldsymbol{\omega}}_{\mathrm{B}} = \mathbf{D}^{+}\Delta\tilde{\boldsymbol{\omega}}_{\mathrm{B}} \ , \tag{2}$$

where [10] further deduces:

$$\mathbf{D}^{+} = -(\mathbf{B}_{\mathrm{BB}}^{-1}\mathbf{B}_{\mathrm{BG}})^{+} = -\mathbf{B}_{\mathrm{BG}}^{+}(\mathbf{B}_{\mathrm{BB}}^{-1})^{-1} = -\mathbf{B}_{\mathrm{BG}}^{+}\mathbf{B}_{\mathrm{BB}} . \tag{3}$$

In the WLS-based estimation, each rotor speed is estimated independently from the others. For the common case of machines connected in antenna, the rotor speed of one synchronous machine can be obtained with only two measures located at the machine bus and its neighbouring bus. The index of the non-zero elements of $\mathbf{D}^{+}$ provide the installation plan to obtain all the rotor speed of synchronous machines with a minimal number of PMUs of a power system.

3) Comparisons of Optimization-based and WLS-based Estimation: Reference [10] discusses advantages and shortcomings of the optimization-based and WLS-based estimation approaches. Relevant conclusions of [10] are as follows:

- The optimization-based estimation approach is robust against bad data, noise and constant data latency, while the WLS-based estimation approach is sensitive to noise and latency. The loss of a machine-bus measure is a critical issue for the WLS-based estimation approach.
- The optimization-based estimation approach has a relevant byproduct, namely the sensitivities of each measure to each estimated variables. Sensitivities cannot be obtained directly from the WLS-based approach.
- The WLS-based estimation approach requires the minimal number of measures. It is, thus, a sort of *decentralized* synchronous machine rotor speed on-line monitoring approach. Moreover, each machine rotor speed estimation is independent from each other. On the other hand, the optimization-based approach is, by construction, a *centralized* method.

Reference [10] discusses the main characteristics of the estimation approaches above, but does not carry out a systematic analysis of the accuracy of such approaches with respect to real-world scenarios.

For example, in [10] it is concluded that, with the same constant measurement delay, the optimization-based estimation presents higher accuracy than the WLS-based one. However, due to their different implementation, the optimization-based estimation is expected to introduce considerably larger measurement delays than the WLS-based approach. According to [11], in fact, PMU measurement delays including the communication latency to control centers can be up to 700 ms, and usually in the range of 100 to 200 ms.

B. Estimation of the Frequency of COI

Since we can estimate the rotor speed of synchronous machines based on the measures of the system bus frequencies, an indirect estimation of the frequency of the COI is also possible. Reference [9] provides the following formula to estimate the frequency of the COI:

$$\omega_{\mathrm{COI}} = \boldsymbol{\xi}^{T}\boldsymbol{\omega}_{\mathrm{B}} + \boldsymbol{\alpha} \ , \tag{4}$$

where $\boldsymbol{\xi}$ is a vector obtained by the product of the vector of normalized inertia of the synchronous machines by $\mathbf{D}^{+}$; and $\boldsymbol{\alpha}$ is an offset vector with $|\boldsymbol{\alpha}| \ll 1$. Mathematical details are given in [9]. $\boldsymbol{\xi}$ and $\boldsymbol{\alpha}$ are piece-wise constant and need to be recomputed only when a topological change occurs.

The most relevant property of the vector $\boldsymbol{\xi}$ is its high sparsity, which descends from the fact that the machines are generally very few with respect to the total number of buses of the system and on the sparsity of $\mathbf{D}^{+}$, whose rationale is duly discussed in [10]. For example, while the AIITS model discussed in [9] has 1,479 buses, only 42 measures are needed to accurately estimate the frequency of COI based on (4).

III. Case Study

The case study considers a dynamic model of the AIITS. The model includes 1,479 buses, 22 conventional synchronous power plants and 176 wind power plants.

All the PMUs installed in the AIITS include a low-pass filter phase-locked loop (LPF-PLL) [25]. The parameters of the LPF-PLL are re-tuned for the different scenarios presented in order to consider the *best-performing* estimation approach in the comparisons. Stochastic wind speeds modelled using Weibull distributions are considered [26]. PMU measurement delays are modelled as a composite of pseudo-periodic, constant and stochastic delays [24], except for the scenario presented in Subsection III-A in which constant delay is assumed. Noise modelled as an Ornstein-Uhlenbeck's process with Gaussian distribution is applied to all bus voltage phase angles [27]. The contingency considered in all scenarios is a three-phase fault occurring at $t = 1$ s and cleared after 150 ms by disconnecting the corresponding line.

All simulations and plots presented in this section were obtained using the software tool Dome [28].

A. Rotor Speed Estimation

In this subsection, the rotor speed of the synchronous machine at bus 1354 is estimated. In [10], the two estimation approaches are compared considering constant measurement delays. The realistic measurement delay, however, is time-varying. To depict the different impacts between both delay models, we compare the WLS-based estimation results with constant delay and realistic-modelled time-varying delay.

Different time-varying PMU measurement delays for the WLS-based and optimization-based estimation approaches are utilised, as shown in Fig. 1. The mean value of the two types of measurement delay are the same: 39.94 ms for the PMU at the neighbouring bus and 20.31 ms for the PMU at the machine bus. Results of the rotor speed estimation are shown in Fig. 2, where $\hat{\omega}_G$ is the actual rotor speed of the generator at bus 1354; $\omega^*_{\rm WLS}$ is the estimation using the WLS-based approach; and $\widetilde{\omega}_{\rm PMU}$ is the PMU measured frequency at Bus 1354.

According to Fig. 2, both scenarios show that WLS-based estimation is still more accurate than the local PMU measures. However, the different models of measurement delays show different impacts on the WLS-based estimation result. The estimation with realistic-modelled delay relatively better tracks dynamic oscillations, but includes high-frequency noise in the signal, while the estimation that utilises constant delay eliminates such a noise but it also reduces the accuracy of the magnitude of the first oscillations. Considering these differences, realistic-modelled time-varying measurement delays are considered in the remainder of this section.

The upper panel of Fig. 3 shows the comparison of WLS-based and optimization-based estimation approaches with the time-varying delay of Fig. 1. From the results, the decentralized WLS-based estimation approach, which introduces shorter measurement delays, presents a slightly better accuracy than the centralized optimization-based estimation. This

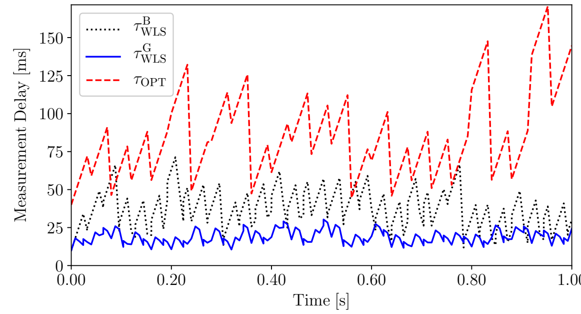

Fig. 1: Trajectories of time-varying delay magnitudes within 1 s of simulation. $\tau^{\rm B}_{\rm WLS}$: delay of the measure at the neighbouring bus of the WLS-based estimation; $\tau^{\rm G}_{\rm WLS}$: delay of the measure at the machine bus of the WLS-based estimation; and $\tau_{\rm OPT}$: typical delay of the optimization-based estimation.

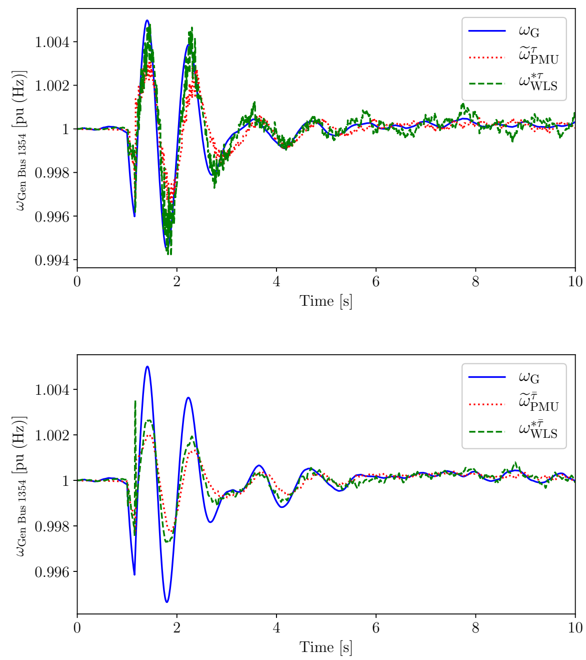

Fig. 2: AIITS undergoing a three-phase fault – estimations of the rotor speed of the synchronous machine at bus 1354. Upper panel: realistic time-varying measurement delay; lower panel: constant measurement delay.

implies that, in real-world applications, without effective delay compensation the WLS-based estimation can be a better option to estimate the rotor speed of synchronous machines.

According to [10], the WLS-based approach cannot accurately estimate the rotor speed without the measure at the machine bus. In real-world power systems, however, it might not be possible to install PMUs at some machine buses due to technical, security and/or ownership issues (this is the case, for example, of the AIITS). With this regard, we consider a scenario for which the measure at the machine bus is not

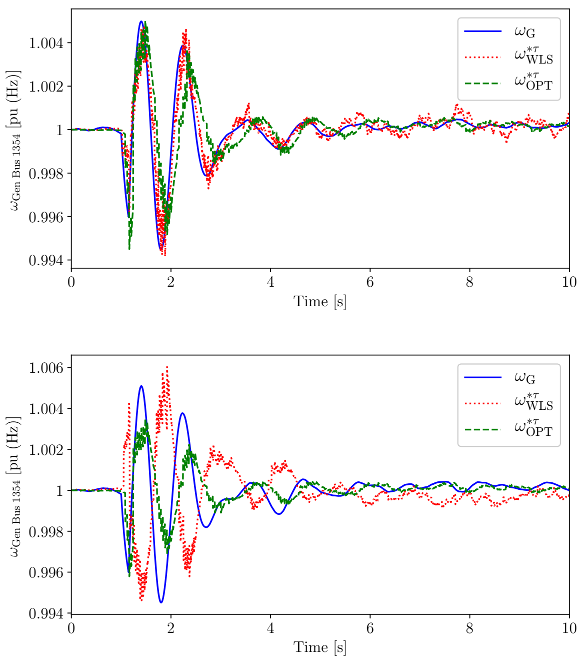

Fig. 3: AIITS undergoing a three-phase fault – estimations of the rotor speed of the synchronous machine at bus 1354. Upper panel: all measures available; lower panel: loss of the measure at machine bus.

available. Results are shown in the lower panel of Fig. 3. The optimization-based approach can effectively estimate the generator rotor speed even without the measure at the machine bus and with the impacts of delay, dynamic wind speed and noise. On the other hand, the WLS-based estimation is no longer reliable.

B. COI Frequency Estimation

This subsection studies the robustness of the approach to estimate the frequency of the COI considering real-world disturbances. As discussed in Section II-B, we consider a maximum of 42 PMUs installed in the AIITS. As the optimization-based estimation approach of the synchronous machine rotor speeds, the estimation of the frequency of the COI is also centralized. The measurement delays of the PMUs in this subsection are thus assumed to be the same as τ_{OPT} in Fig. 1.

A three-phase fault is simulated in the North-Ireland transmission system, which is one of the 20 sub-area systems of the AIITS, and includes 5 synchronous power plants and 10 PMUs. We consider a critical scenario where the fault results in the loss of all the PMUs of the North-Ireland sub-area system due to protection tripping. Figure 4 compares the estimation results of this scenario with the scenario where all measures are available. Figure 4 also includes the frequency measured at a pilot bus to account for the common practice of TSOs to estimate the frequency of the COI.

The estimation approach (4) with 42 PMUs accurately tracks the dynamic behaviour of the frequency of the COI

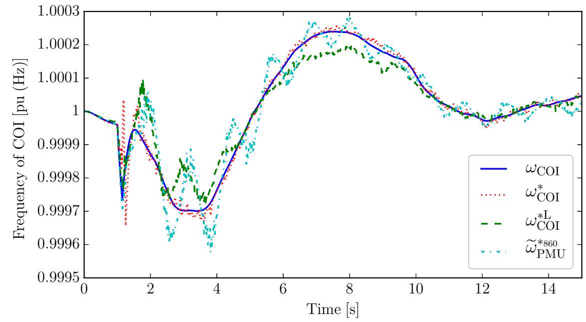

Fig. 4: AIITS undergoing a three-phase fault – estimations of the frequency of the COI. $\hat{\omega}_{\text{COI}}$: actual frequency of the COI; ω^{*}_{COI}: eq. (4) with all measures available; $\omega^{*\text{L}}_{\text{COI}}$: eq. (4) with loss of the measures of the North-Ireland sub-area system; $\widetilde{\omega}_{\text{PMU}}$: measured frequency at the pilot bus.

following the fault clearance. The accuracy of this estimation approach decreases when losing the 10 PMUs of the North-Ireland system, but it nevertheless shows a good robustness against the loss of about 25% of the total number of measures. The worst estimation is obtained when measuring the frequency at the pilot bus, as it captures not only the overall trend of the frequency of the COI, but also some local oscillatory modes that are naturally filtered out by the COI.

The last example presented in this section considers the estimation of the frequency of the COI of a sub-area system. With this aim, the North-Ireland sub-area transmission system of the AIITS is considered. In this sub-area system, the synchronous power plant at bus 1236 provides the highest inertia, which represents 34.54% of the total. Bus 1236 is thus chosen as the pilot bus for the comparison.

The results of the estimation comparison of the sub-area COI frequency are shown in the upper panel of Fig. 5. The estimation based on (4) and the pilot bus measure have similar accuracies. The scenario where a PMU cannot be installed at the machine bus 1236 is also studied. In this scenario, the pilot bus is chosen as the machine neighbouring bus 1237, and results are shown in the lower panel of Fig. 5. The comparison indicates that the approach of (4) is still accurate even if the measure at the bus where the machine with the highest inertia of the sub-area system is not available, despite the phase-shift introduced in the estimation.

IV. Conclusion

This paper studies the accuracy of several PMU-based online estimation approaches for synchronous machine rotor speeds and the frequency of COI. The comparison is based on the 1,479-bus model of the all-island Irish transmission system with inclusion of stochastic wind speed variations, and PMU measurement delays and noise. As opposed to previous studies which show that more measures imply better accuracies in the frequency estimation, this paper demonstrates that, in real-world applications, factors such as PMU measurement delays

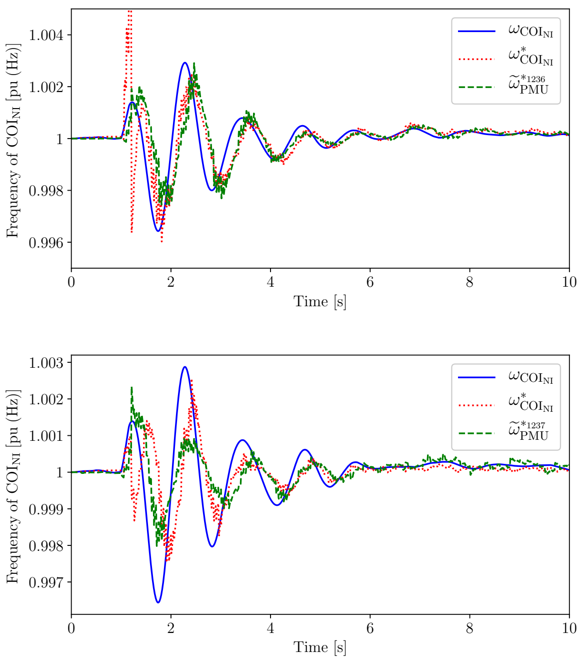

Fig. 5: AIITS undergoing a three-phase fault – estimations of the frequency of the COI of the North-Ireland sub-area system. Upper panel: all measures available; lower panel: loss of measure at bus 1236. $\omega_{\mathrm{COI_{NI}}}$: actual frequency of the sub-area COI; $\omega^{*}_{\mathrm{COI_{NI}}}$: estimation by means of eq. (4); and $\widetilde{\omega}^{1236(1237)}_{\mathrm{PMU}}$: frequency measured at bus 1236 (1237).

and system size and/or topology, might lead to the break-even point where the accuracy is compromised if too many measures are used. TSOs should always choose estimation approaches with the full considerations of power system features and the potential impact from measurement delays to fulfil the required accuracy with fewer measures.

References

[1] L. Cheng, G. Chen, W. Gao, F. Zhang, and G. Li, "Adaptive time delay compensator (ATDC) design for wide-area power system stabilizer," *IEEE Trans. on Smart Grid*, vol. 5, pp. 2957–2966, Nov 2014.

[2] R. Hadidi and B. Jeyasurya, "Reinforcement learning based real-time wide-area stabilizing control agents to enhance power system stability," *IEEE Trans. on Smart Grid*, vol. 4, pp. 489–497, March 2013.

[3] A. K. Singh and B. C. Pal, "Decentralized dynamic state estimation in power systems using unscented transformation," *IEEE Trans. on Power Systems*, vol. 29, pp. 794–804, March 2014.

[4] N. Zhou, D. Meng, and S. Lu, "Estimation of the dynamic states of synchronous machines using an extended particle filter," *IEEE Trans. on Power Systems*, vol. 28, pp. 4152–4161, Nov 2013.

[5] E. Ghahremani and I. Kamwa, "Online state estimation of a synchronous generator using unscented Kalman filter from phasor measurements units," *IEEE Trans on Energy Conversion*, vol. 26, pp. 1099–1108, Dec 2011.

[6] C. J. Tavora and O. J. M. Smith, "Characterization of equilibrium and stability in power systems," *IEEE Transactions on Power Apparatus and Systems*, vol. PAS-91, pp. 1127–1130, May 1972.

[7] D. Fabozzi and T. V. Cutsem, "On angle references in long-term time-domain simulations," *IEEE Transactions on Power Systems*, vol. 26, pp. 483–484, Feb 2011.

[8] F. Milano and Á. Ortega, "Frequency divider," *IEEE Trans. on Power Systems*, vol. 32, pp. 1493–1501, March 2017.

[9] F. Milano, "Rotor speed-free estimation of the frequency of the center of inertia," *IEEE Trans. on Power Systems*, vol. 33, pp. 1153–1155, Jan 2018.

[10] F. Milano, Á. Ortega, and A. J. Conejo, "Model-agnostic linear estimation of generator rotor speeds based on phasor measurement units," *IEEE Trans. on Power Systems*, pp. 1–1, 2018.

[11] V. Terzija, G. Valverde, D. Cai, P. Regulski, V. Madani, J. Fitch, S. Skok, M. M. Begovic, and A. Phadke, "Wide-area monitoring, protection, and control of future electric power networks," *Procs of the IEEE*, vol. 99, pp. 80–93, Jan 2011.

[12] E. Farantatos, G. K. Stefopoulos, G. J. Cokkinides, and A. P. Meliopoulos, "PMU-based dynamic state estimation for electric power systems," in *Procs of the IEEE PES General Meeting*, pp. 1–8, July 2009.

[13] A. K. Singh and B. C. Pal, "Decentralized dynamic state estimation in power systems using unscented transformation," *IEEE Trans. on Power Systems*, vol. 29, pp. 794–804, March 2014.

[14] N. Zhou, D. Meng, Z. Huang, and G. Welch, "Dynamic state estimation of a synchronous machine using PMU data: A comparative study," *IEEE Trans. on Smart Grid*, vol. 6, pp. 450–460, Jan 2015.

[15] E. Ghahremani and I. Kamwa, "Local and wide-area PMU-based decentralized dynamic state estimation in multi-machine power systems," *IEEE Trans. on Power Systems*, vol. 31, pp. 547–562, Jan 2016.

[16] A. Rouhani and A. Abur, "Linear phasor estimator assisted dynamic state estimation," *IEEE Trans. on Smart Grid*, vol. 9, pp. 211–219, Jan 2018.

[17] I. Kamwa, R. Grondin, and Y. Hebert, "Wide-area measurement based stabilizing control of large power systems-a decentralized/hierarchical approach," *IEEE Trans. on Power Systems*, vol. 16, pp. 136–153, Feb 2001.

[18] B. Naduvathuparambil, M. C. Valenti, and A. Feliachi, "Communication delays in wide area measurement systems," in *Procs of the 34th Southeastern Symposium on System Theory*, (Huntsville, LA, USA), Mar. 2002.

[19] S. Wang, X. Meng, and T. Chen, "Wide-area control of power systems through delayed network communication," *IEEE Trans. on Control Systems Technology*, vol. 20, pp. 495–503, Mar. 2012.

[20] W. Yao, L. Jiang, J. Wen, Q. H. Wu, and S. Cheng, "Wide-area damping controller of FACTS devices for inter-area oscillations considering communication time delays," *IEEE Trans. on Power Systems*, vol. 29, pp. 318–329, Jan 2014.

[21] D. Dotta, A. S. e Silva, and I. C. Decker, "Wide-area measurements-based two-level control design considering signal transmission delay," *IEEE Trans. on Power Systems*, vol. 24, pp. 208–216, Feb 2009.

[22] J. W. Stahlhut, T. J. Browne, and G. T. Heydt, "Latency viewed as a stochastic process and its impact on wide area power system control signals," *IEEE Trans. on Power Systems*, vol. 23, pp. 84–91, Feb. 2008.

[23] M. Liu and F. Milano, "Small-signal stability analysis of power systems with inclusion of periodic time-varying delays," in *Procs of the 20th Power System Computation Conference*, (Dublin, Ireland), June 2018.

[24] M. Liu, I. Dassios, G. Tzounas, and F. Milano, "Stability analysis of power systems with inclusion of realistic-modeling of wams delays," *IEEE Trans. on Power Systems*, pp. 1–1, 2018.

[25] Á. Ortega and F. Milano, "Comparison of bus frequency estimators for power system transient stability analysis," in *Procs of the IEEE International Conference on Power System Technology (POWERCON)*, pp. 1–6, Sept 2016.

[26] F. Milano, *Power System Modelling and Scripting*. London: Springer, 2010.

[27] F. Milano and R. Zárate-Miñano, "A systematic method to model power systems as stochastic differential algebraic equations," *IEEE Transactions on Power Systems*, vol. 28, pp. 4537–4544, Nov 2013.

[28] F. Milano, "A Python-based software tool for power system analysis," in *Procs of the IEEE PES General Meeting*, (Vancouver, BC), July 2013.