On the Dynamic Coupling of the Autocorrelation of Stochastic Processes and the Standard Deviation of the Trajectories of Power System Variables

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***Abstract*—The paper originates from the observation that, due to the nonlinearity of the power system model, the autocorrelation of noise impacts on the standard deviations of the trajectories of the variables of the system in an unpredictable way. A case study based on the well-known two-area system with inclusion of stochastic load models is presented. Simulation results show that an increase of the autocorrelation of load consumption causes a significant increase of the standard deviation of the active and reactive powers generated by the synchronous machines. In some scenarios, the increase of the autocorrelation gives rise to instability and drives the system to voltage collapse.**

***Index Terms*—Stochastic processes, autocorrelation, standard deviation, volatility, transient stability.**

I. Introduction

Several studies based on real-world measurements have analyzed the short-term dynamic behavior of stochastic processes such as load consumption and renewable generation and concluded that such a behavior can be modeled as a set of stochastic differential equations [1], [2]. It has also been observed that stochastic processes are characterized by an exponentially decaying autocorrelation function independent of the probability distribution of the process [3]. This paper focuses on evaluating the impact of autocorrelation of stochastic processes on the dynamic behavior and transient stability of power systems.

Load consumption and renewable energy resources are sources of uncertainty and volatility. For the long term analysis, the availability of a large amount of historical data has led to relatively accurate load and renewable generation forecasting. In short-term studies, however, the randomness of stochastic fluctuations cannot be fully anticipated. One of the most relevant approaches for the modeling of short-term variations is thus through stochastic processes [1]–[7]. In particular, continuous fluctuations can be formulated as stochastic differential equations (SDEs) that contain two terms, namely, *drift* and *diffusion* [8]. The drift term is the deterministic part of an SDE and defines the autocorrelation of the process, i.e. its "evolution in time." Whereas the diffusion term is the non-deterministic part of an SDE and defines the standard deviation of the stochastic process in stationary conditions.

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The overall dynamic behavior of a power system subject to stochastic disturbances can be modeled as a set of SDEs. Several analytical solutions are available to solve SDE models and calculate the probability distribution of stability of the power system. Analytical solutions are formulated on the bases of the theory of stability of SDEs, which only work for linear systems [9], [10] and/or highly simplified models [11], [12]. Since these methods require strong simplifications and/or linearization of the power system equations, they may fail to capture the impact of stochastic processes for large disturbances and the dynamic coupling arising from an increasing size and complexity of the power system model. More importantly for the study considered in this paper, existing studies focus on stationary conditions, i.e. consider the probability distribution and standard deviation of the variables. However, the autocorrelation of the stochastic processes and its impact on short-term dynamics is often neglected.

In this paper, no simplification or linearization is applied to the model. The power system is formulated as a set of non-linear stochastic differential-algebraic equations (SDAEs) and provides a detailed model of the transient dynamic behavior of the power system subject to stochastic disturbances [4], [13], [14]. Reference [4], in particular, provides a generalized and systematic modeling approach. Hence, it does not depend on the size or complexity of the power system to model power systems subject to stochastic disturbances as a set of non-linear SDAEs. A byproduct of this modeling approach, however, is that no analytical solutions, of the resulting SDAEs that describe the power system model, are available. Hence, we rely on numerical methods such as trapezoidal integration method for the deterministic differential-algebraic equations (DAEs), and Euler-Maruyama method for stochastic differential equations, are utilized to integrate the non-linear SDAEs [4].

References [15]–[17] employed Monte Carlo time domain simulation (TDS) to evaluate the probability distribution of transient stability of power system with inclusion of stochastic disturbances. In [18], the authors improved the computation time of the Monte Carlo TDS by utilizing importance sampling method. In [19], a method was proposed to properly initialize SDAEs to reduce execution time of Monte Carlo TDS. The effect of correlation between active and reactive load power consumption on the voltage stability of the power system utilizing Monte Carlo TDS was studied in [20]. Similarly, in this paper, Monte Carlo TDS are carried out to extract meaningful statistical properties, such as the standard deviation and the autocorrelation of the trajectories of relevant variables.

The vast majority of literature on SDEs or SDAEs evaluates the probability distribution of the stability of power systems subject to stochastic disturbances based on the probability distribution of the input stochastic processes. Some exceptions are [2] and [3], which propose methods to synthesize SDEs with given autocorrelation and probability distribution. However, the question of what is the effect of the autocorrelation of a stochastic process on the stability of the power system still remains unanswered.

This paper focuses on the dynamic effect of the autocorrelation, which is a property of stochastic processes, on the transient stability of power systems.

Specific contributions are as follows.

- The paper shows the coupled effect of the drift and diffusion terms of the stochastic processes. In particular, the paper shows that, in the short term, increasing the autocorrelation of a stochastic process leads to reach faster stationary conditions, e.g., constant standard deviation.
- A byproduct of the observation above is that processes with same standard deviation but different autocorrelations have a different dynamic impact on the system. The paper shows that this impact is ultimately affecting the standard deviation of the variables of the system. In other words, due to nonlinearity, the dynamics of the drifts of the stochastic processes plays an important role on the stationary conditions of the overall system.
- Finally, the paper shows that processes with low standard deviation but high autocorrelation can drive the system to instability.

The remainder of the paper is organized as follows. Section II introduces the non-linear stochastic differential-algebraic equations, which are utilized to model power system. Section III introduces and explains the volatility model, which is an Ornstein-Uhlenbeck's process. Section IV presents the simulation results obtained by simulating the dynamic model of a 2-area system as a set of non-linear SDAEs and analyzes different results obtained for different autocorrelation of the stochastic processes. Finally, Section V draws conclusions.

II. Modeling

The dynamic behavior of the power system is conventionally modeled as a set of non-linear DAEs [21], as follows:

$$\begin{aligned} \dot{\boldsymbol{x}} &= \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{y}) \,, \\ \boldsymbol{0} &= \boldsymbol{g}(\boldsymbol{x}, \boldsymbol{y}) \,, \end{aligned} \tag{1}$$

where $\boldsymbol{f} : \mathbb{R}^{l+m+n} \mapsto \mathbb{R}^m$ are the differential equations; $\boldsymbol{g} : \mathbb{R}^{l+m+n} \mapsto \mathbb{R}^l$ are the algebraic equations; $\boldsymbol{x} \in \mathbb{R}^l$ is a vector of state variables; $\boldsymbol{y} \in \mathbb{R}^m$ is a vector of algebraic variables.

The effect of randomness volatility is modeled by introducing SDEs into deterministic DAEs [4], [13], [14], as follows:

$$\begin{aligned} \dot{\boldsymbol{x}} &= \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{\kappa}) \,, \\ \boldsymbol{0} &= \boldsymbol{g}(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{\kappa}) \,, \\ \dot{\boldsymbol{\kappa}} &= \boldsymbol{a}(\boldsymbol{\kappa}) + \boldsymbol{b}(\boldsymbol{\kappa}) \circ \boldsymbol{\xi} \,, \end{aligned} \tag{2}$$

where $\boldsymbol{\kappa} \in \mathbb{R}^n$ represents the vector of uncorrelated stochastic processes; $\boldsymbol{a} : \mathbb{R}^n \mapsto \mathbb{R}^n$ and $\boldsymbol{b} : \mathbb{R}^n \mapsto \mathbb{R}^n$ are the *drift*, and *diffusion* terms, respectively. If the drift is a vector of linear functions, e.g. $\boldsymbol{a}(\boldsymbol{\kappa}) = \boldsymbol{\alpha} \circ \boldsymbol{\kappa}$, the elements of the vector $\boldsymbol{\alpha}$ are called *autocorrelation coefficients*. $\circ$ represents the Hadamard product, i.e. the element-by-element product of two vectors. Finally, $\boldsymbol{\xi} \in \mathbb{R}^n$ is a vector of n-dimensional independent *Gaussian white noise*, which is the formal representation of time derivative of the Wiener process.

A Wiener process W_i is a time continuous stochastic process with Gaussian distribution and the following properties:

1) $W_i(0) \equiv 0$.
2) $W_i(t)$ is a continuous function of t.
3) Increments of $W_i(t)$ follow Gaussian distribution, i.e. $\forall t \geq 0$, $dW_i = W_i(t+h) - W_i(t) \sim \mathcal{N}(0, h)$ where $\mathcal{N}(\mu, \sigma^2)$ represents Gaussian distribution with mean μ and standard deviation σ.
4) $W_i(t)$ has independent increments, i.e. $\mathrm{cov}[dW_i, dW_j] = 0$, where $i \neq j$, and dW_i is the i_{th} increment of $W_i(t)$.

A. Stochastic Load Model

For simplicity but without loss of generality, this paper focuses on the stochastic disturbances introduced into the power system by stochastic loads. With this regard, we use a well-known stochastic load model [4], [12]–[14], which has been derived from a voltage dependent load model, as follows:

$$\begin{aligned} p_{\mathrm{L}}(t) &= (p_{\mathrm{L}0} + \kappa_p(t))(v(t)/v_0)^{\gamma} \,, \\ q_{\mathrm{L}}(t) &= (q_{\mathrm{L}0} + \kappa_q(t))(v(t)/v_0)^{\gamma} \,, \\ \dot{\kappa}_p(t) &= a_p(\kappa_p(t)) + b_p(\kappa_p(t))\, \xi_p(t) \,, \\ \dot{\kappa}_q(t) &= a_q(\kappa_q(t)) + b_q(\kappa_q(t))\, \xi_q(t) \,. \end{aligned} \tag{3}$$

where $p_{\mathrm{L}0}$ and $q_{\mathrm{L}0}$ are the mean values (for the duration of TDS) of active and reactive power consumption, respectively; $v(t)$ is the magnitude of the bus voltage at the load bus; v_0 is the initial value of this voltage magnitude at the start of TDS; and γ classifies load as constant power load for $\gamma = 0$ and constant impedance load for $\gamma = 2$; and a, b, and ξ have the same meaning as in (2). The terms a and b depend on the type of stochastic process being used to model volatility. In this paper we choose an Ornstein-Uhlenbeck's process (OUP) to model volatility.

III. Ornstein-Uhlenbeck's Process

An OUP is a linear implementation of the SDE introduced in (2). Hence, both drift and diffusion terms can independently modify the dynamic behavior of an OUP.

The OUP is mean-reverting, i.e. it always tends to its mean value and it shows constant standard deviation in stationary conditions. These features make the OUP adequate to model the volatility in physical processes such as stochastic load dynamics [1], [5], [6] and short-term wind fluctuations [2], [22], [23]. An OUP is defined as:

$$\dot{\kappa} = -\alpha(\kappa - \mu) + \beta\xi \,, \tag{4}$$

where α is the autocorrelation coefficient; β is the coefficient of the diffusion term; μ is the mean value; and ξ is the white noise. The process resulting from (4) is a real-valued process that follows a Gaussian probability distribution given by $\mathcal{N}(\mu, \sigma^2)$, and $\beta = \sigma\sqrt{2\alpha}$.

An OUP defined in (4) is a linear combination of two terms: drift and diffusion. Hence, the drift and diffusion of an OUP can be adjusted independently. As a result, OUPs with different values of autocorrelation and, hence, different dynamic behavior, can have same probability distribution in stationary conditions. In fact, the probability distribution function of (4) is:

$$P(\kappa) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\kappa-\mu}{\sigma}\right)^2} \,, \tag{5}$$

which does not depend on α.

A. Illustrative Example

In this paper, the goal is to analyze the impact of autocorrelation of a stochastic process on the stability of the power systems utilizing numerical TDS. With this aim, this section considers an illustrative example.

Table I shows a set of parameters of OUPs. Figure 1 illustrates the time series of OUPs generated from the parameters in Table I. It is important to note that the processes shown in Fig. 1 (top panel) have the same probability distribution in stationary conditions. However, their transient behavior is significantly different because of the different value of α and, hence, of their autocorrelation. On the other hand, the bottom panel of Fig. 1 shows OUPs generated with different values of σ but same values of α. Comparing the upper and lower panel, it is evident that, from the dynamic point of view, a process with high autocorrelation and low standard deviation has a similar effect as a process with low autocorrelation and high standard deviation.

Formally, the autocorrelation of a stochastic process is defined as the measure of correlation of the present value to the past and future values:

$$R(\tau) = \frac{E[(\kappa_t - \mu)(\kappa_{t+\tau} - \mu)]}{\sigma^2} \,, \tag{6}$$

where E is the expectation operator; κ_t is the value of the process at time t; and τ is the time lag.

Figure 2 illustrates the autocorrelations of the OUPs shown in Fig. 1. The autocorrelation is always equal to 1 for $\tau = 0$, by definition. As τ increases the correlation of the OUPs

TABLE I: Parameters of Ornstein-Uhlenbeck's processes.

	κ_1	κ_2	κ_3	κ_4	κ_5	κ_6
α	1	0.1	0.01	0.01	0.01	0.01
σ	0.1	0.1	0.1	0.4	0.3	0.2

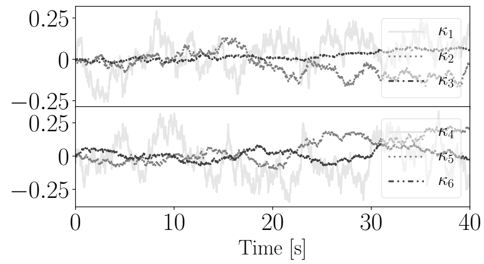

Fig. 1: Time domain profile of Ornstein-Uhlenbeck's processes; the values of parameters can be found in Table I.

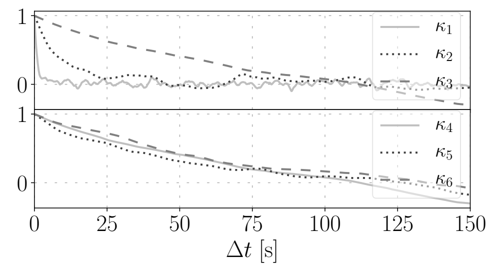

Fig. 2: Exponentially decaying autocorrelation of Ornstein-Uhlenbeck's processes; the values of parameters can be found in Table I.

between current and future values decreases exponentially and decreases the faster the higher the value of α. This exponentially decaying autocorrelation is observed in several physical processes such as stochastic load dynamics [1], [5] and wind fluctuations [2], [22]. Note, however, that processes with different σ and same α show similar time evolution of the autocorrelation (see bottom panel of Fig. 2).

It is interesting to note that, taken alone, neither the time series of the OUPS shown in Fig. 1 nor the dynamic behavior of the autocorrelation shown in Fig. 2 allow to distinguish between the OUPs. A more effective way to visualize the behavior of stochastic processes is through a Monte Carlo TDS analysis. With this aim, 1000 trajectories of each process of Table I, with initial condition $\kappa_i(0) = 0$, and a time step $h = 0.01$ s for the increments of the Wiener process are simulated. The standard deviation of all the trajectories for each process is calculated at every time step and plotted against time in Fig. 3. The results shown in Fig. 3 indicate that the time at which a stochastic process becomes stationary, i.e. constant standard deviation, depends only on the autocorrelation of the process. On the other hand, the spread of the trajectories in stationary conditions depends only on the value of the standard deviation.

So far, we have considered independent OUPs. In the SDAE model (2), however, the OUPs are dynamically coupled with the rest of the system. Common sense would suggest that the autocorrelation of the OUPs affects exclusively the transient, while the standard deviation affects only the stationary conditions. However, since the variables $\boldsymbol{\kappa}$ appear in the nonlinear DAEs, this intuition is not correct. The remainder of this paper shows that the autocorrelation of the OUPs also impacts on the stationary conditions of the system.

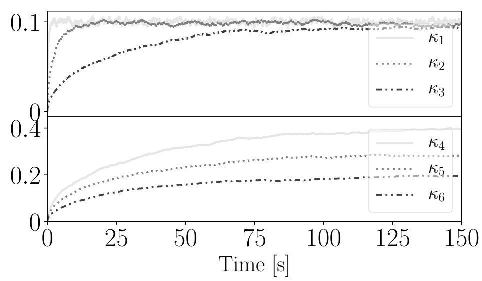

Fig. 3: Standard deviation of realizations of Ornstein-Uhlenbeck's processes; the values of parameters can be found in Table I.

IV. Case Study

This section presents the simulation results of Monte Carlo TDS of the non-linear SDAEs using a dynamic model of the well-known two-area system [21], which consists of 11 buses, 12 lines/transformers, and 4 synchronous generators that are modeled through a VI-order model and are equipped with IEEE ST1a exciters, turbine governors, and an AGC to ensure a secure operation of the grid. The system base is 100 MVA. Stochastic disturbances are included in the load models as discussed in Section II-A.

The impact of the autocorrelation of stochastic processes on the dynamic behavior of the power system is evaluated by considering the evolution in time of the standard deviation of some relevant variables of the system. With this aim, Table II defines six cases with various combinations of α and σ of the OUPs that describe the loads.

The simulations are performed utilizing Monte Carlo TDS. 1000 trajectories are simulated for each case in Table II. The non-deterministic part of non-linear SDAEs is solved by Euler scheme utilizing a time step of $h = 0.01$ s, whereas the deterministic part is solved by employing implicit-trapezoidal integration scheme with a step size of $\Delta t = 0.01$ s. The total simulated time for each trajectory is $t = 150$ s. The results presented in this section have been verified with other numerical schemes. This allow concluding that the instabilities observed for some scenarios are in effect due to the actual behavior of the system and are not due to numerical issues.

Figures 4 and 5 show the evolution of the standard deviation of active power generation p_g for cases $2a$, $2b$, and $2c$. It is important to note that active powers p_g belong to the vector of algebraic variable $\boldsymbol{y}$ of (2), i.e. their stochastic behavior is the result of the inclusion in $\boldsymbol{f}$ and $\boldsymbol{g}$ of the stochastic variable $\boldsymbol{\kappa}$. Figures 4 and 5 show that the standard deviation of the variables increases as the autocorrelation of the stochastic processes increases. This occurs despite the fact that the processes have the same probability distribution in stationary conditions, as shown from Fig. 3. Note also that all the cases presented in Figs. 4 and 5 reach the same mean value.

The values of standard deviation of p_g as well as of the reactive power q_g of all generators at stationary conditions are shown in Tables III and IV. The results in both tables indicate that the value of σ_{p_g} and σ_{q_g} increases from 85% to 330%, as α is increased from 0.01 to 1 s^{-1}.

TABLE II: Autocorrelation α and standard deviation σ of stochastic load consumption for different cases.

cases	α [s^{-1}]	σ_p [% of p_{L0}]	σ_q [% of q_{L0}]
case $1a$	0.01	0.4	0.4
case $1b$	0.1	0.4	0.4
case $1c$	1	0.4	0.4
case $2a$	0.01	0.6	0.6
case $2b$	0.1	0.6	0.6
case $2c$	1	0.6	0.6

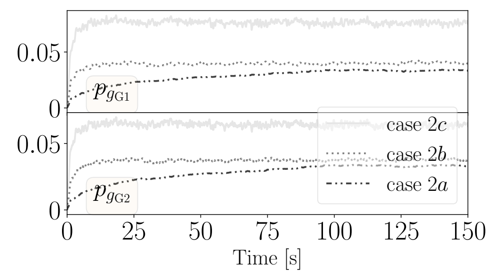

Fig. 4: Standard deviation of $p_{g\mathrm{G1}}$ and $p_{g\mathrm{G2}}$ for cases $2a$, $2b$, and $2c$.

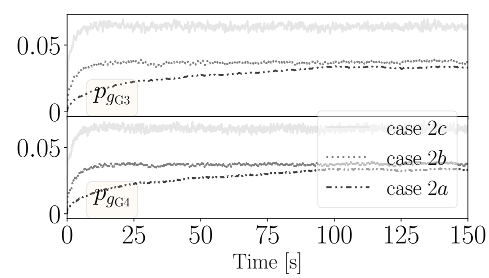

Fig. 5: Standard deviation of $p_{g\mathrm{G3}}$ and $p_{g\mathrm{G4}}$ for cases $2a$, $2b$, and $2c$.

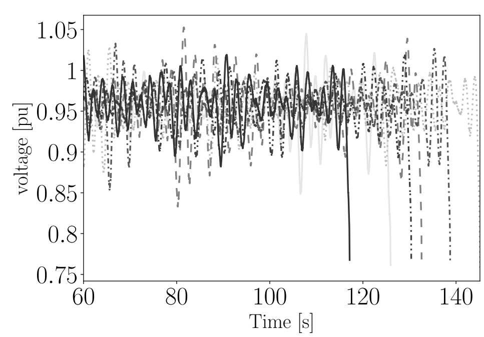

Fig. 6: Voltage profile at bus 8 for a few unstable trajectories taken from case $2c$.

It is also interesting to note that 197 simulations are unstable for case $2c$. For illustration, a selection of the trajectories of the voltage magnitude at Bus 8 for case $2c$ are shown in Fig. 6. On the other hand, no instability occurs for cases $2a$ and $2b$. These results indicate that the autocorrelations of stochastic processes, not the standard deviations and probability distributions alone, are crucial parameters for the stability analysis of power systems. In fact, high standard deviations might not be dangerous for the system if the autocorrelation is sufficiently low. On the other hand, if the autocorrelations of the processes are sufficiently high, even if their standard deviations are low, instability can occur.

TABLE III: Standard deviation of active and reactive power generation of synchronous generators for the three scenarios simulating stochastic loads for cases $1a$, $1b$, and $1c$.

Standard deviation	case $1a$ absolute [pu]	case $1b$ % increase[a]	case $1c$ % increase[a]
$p_{g_{G1}}$	0.022211	19.01	120.69
$p_{g_{G2}}$	0.022017	13.4	88.13
$p_{g_{G3}}$	0.022033	11.73	84.69
$p_{g_{G4}}$	0.022031	12.55	89.27
$q_{g_{G1}}$	0.030829	70.12	331.42
$q_{g_{G2}}$	0.048432	45.05	238.81
$q_{g_{G3}}$	0.032359	52.77	256.9
$q_{g_{G4}}$	0.053047	31.85	178.36

[a]Note: % increase is calculated based on case $1a$.

TABLE IV: Standard deviation of active and reactive power generation of synchronous generators for the three scenarios simulating stochastic loads for cases $2a$, $2b$, and $2c$.

Standard deviation	case $2a$ absolute [pu]	case $2b$ % increase[a]	case $2c$ % increase[a]
$p_{g_{G1}}$	0.03343	20.44	126.88
$p_{g_{G2}}$	0.03313	14.9	92.77
$p_{g_{G3}}$	0.03313	13.23	92.8
$p_{g_{G4}}$	0.03313	14.03	94.33
$q_{g_{G1}}$	0.04608	74.8	353.61
$q_{g_{G2}}$	0.07215	49.79	257.51
$q_{g_{G3}}$	0.04834	57.42	277.62
$q_{g_{G4}}$	0.07903	36.38	194.92

[a]Note: % increase is calculated based on case $2a$.

V. Conclusions

This work evaluates the impact of the autocorrelation of stochastic load consumption on the transient stability of the power system and on the standard deviation of the trajectories of system variables such as the output powers of synchronous generators.

The case study shows that the autocorrelation of the stochastic processes impacts significantly on the standard deviation of the system variables. In fact, a high value of the autocorrelation of a stochastic process, even if its standard deviations is small and acceptable in stationary conditions, can lead to high variations the system variables and, in some cases, to instability. This non-intuitive result is due to the dynamic coupling of the autocorrelation of stochastic processes with the nonlinearity of the SDAEs that define the power system model. We believe that this is an important result that system operators should take into account when carrying out the stability analysis of their grids.

Future work will focus on the impact of the autocorrelation of stochastic renewable energy sources as well as on the definition of techniques able to reduce the impact and/or the coupling of autocorrelation on the dynamic behavior of the power system.

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