Structured Model Reduction of Power Systems

Christopher Sturk, Luigi Vanfretti, Federico Milano, and Henrik Sandberg

Abstract— This paper shows how structured model order reduction can be applied to power systems. For power systems divided into a study area and an external area, the proposed algorithm can be used to reduce the external area to a low order linear system, while retaining the nonlinear description of the study system. The reduction of the external area is done in such a way that the study system is affected as little as possible. It is shown that a lower model order can be attained when information about the study system is taken into consideration, than if the external system is reduced independently of it.

Index Terms— Model reduction of power systems, internal systems, structured model reduction

I. INTRODUCTION

Power systems today are colossal networks of interconnected power apparatus often spanning large geographical areas. With the growing trend in facilitating additional interconnections to neighbouring systems, the size and complexity of these networks will likely continue to increase, and thus, bring challenges for planning, operations and control of these networks. Hence, reduced-order models of power systems are desirable for many applications and studies, particularly for fast and cost-efficient stability analysis. Research on power system model reduction is extensive, and some methodologies focusing on specific applications have been implemented in software for automated model reduction. Generally speaking, model reduction of large power networks, typically known as *Power System Dynamic Equivalencing* [1], has the main aim of providing a system equivalent model able to reproduce the aggregated steady-state [2] and dynamic characteristics of the full-order network [3], while at the same time being compatible with the available computation tools for power system analysis [4], [5].

Model reduction techniques are often developed for specific applications such as: steady-state power flow for security assessment [6], small-signal stability analysis [7], transient stability [8], [9], network equivalents for electromagnetic transient studies [10], and control design applications [11]. In these applications the power system is divided into

an *internal* (or study system) and an *external* system. The internal or study system is usually set by the boundaries of the part of the system that a specific utility owns or it may be the area in which a particular power market is defined, usually this portion of the network is *electrically close*. However, it would be advantageous to arbitrarily set the area corresponding to the study system without any restrictions, which could aid in the design of power plant controllers and coordination of system protections. The external system is usually electrically distant, and in practice it is used to denote the service area of other utilities. The internal system is usually represented in detail while a reduced model is used for the external system.

From these approaches, there is particular interest in those that can preserve network structures and can be used for small signal analysis, transient stability analysis, and control design. Coherency based methodologies [1], [9], [12], [13] have been well accepted by the power engineering community and automated software for their application exist [4]. These methods start by identifying coherency in the generators of the power system [14], and in a second stage to proceed with the dynamic reduction of the system. The dynamic reduction process itself is carried out by aggregating the network [15] and aggregating the generators [16], [13]. At a later stage, it may even be possible to aggregate excitation controllers [17]. Although coherency methods have been accepted as the most reliable for power system dynamic equivalencing, the major drawback is that it may not always be possible to reduce specific parts of the power network, the nature of coherency is to cluster generator groups which imposes the areas in which the network can be divided. To overcome this limitation, some approaches that are capable of retaining a part of the network have been proposed [18], [19]

With the recent developments in power systems an increased interest has been seen from the automatic control community. Among other things it has been shown how model reduction algorithms popular in control can be applicable. These algorithms typically have a strong theoretical foundation and they are also very general in the sense that they are not targeted to a particular application. This makes them a good candidate for the reduction of power systems composed not only of synchronous machines but also of for instance renewable energy sources.

In [20] it is shown how Krylov subspace model reduction can be applied to the external area before reconnection with the study area. Principal component analysis (PCA) is another popular model reduction algorithm. In particular it is well-suited for reduction of nonlinear systems. In [21] it

was used to project the states of the external system onto a subspace of lower dimension.

What we propose in this paper is to apply structured model order reduction to power systems with an external and a study area. The idea is to reduce the external system while trying to minimize the effect it will have on the study area. This is the main objective of structured model reduction, namely to reduce models locally while ensuring a small global model error.

Model reduction where various structural constraints are taken into account ("structured model reduction") has been considered in several papers. For example, in [22] frequency-weighted model reduction problems are considered, and in [23] controller reduction is considered. More general inter-connection structures have been considered in, for example, [24], [25], [26].

The structure of the paper is as follows: In Section II we formulate the problem we want to solve. Section III summarizes the theory of structured model order reduction and in Section IV it is shown how this theory can be applied to power systems. Section V introduces the WSCC 3-machine, 9-bus system to which the proposed model reduction algorithm is applied.

II. PROBLEM FORMULATION

We will assume that the power system we want to reduce can be divided into one part with system variables of interest to us, called the study area, and another part called the external area, that is only of interest in terms of its effect on the system variables in the study area (Fig. 1). This is motivated by the fact that one often only focuses on parts of the power system. Under this assumption the aim is to reduce the external area to a linear time-invariant (LTI) system of lower order, while retaining the nonlinear description of the study area. This is relevant since it allows for a physical interpretation of the reduced power system, which is helpful if one wants to simulate changes to the study area in terms of for instance transmission line failures or for the purpose of nonlinear control design.
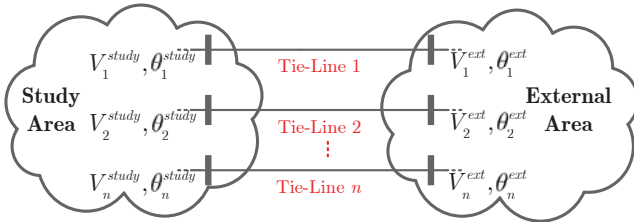

Fig. 1. The power system is divided into a study area and an external area, which is to be reduced.

The interface between the systems is defined by their n tie-lines and buses. Given that the network with N buses has an admittance matrix $Y = G + jB$, every bus satisfies the two power equations

$$\tilde{P}_i = V_i^2 G_{ii} + \sum_{j=1;j\neq i}^{N} V_i V_j B_{ij} \sin(\theta_i - \theta_j)$$
$$+ \sum_{j=1;j\neq i}^{N} V_i V_j G_{ij} \cos(\theta_i - \theta_j)$$
$$\tilde{Q}_i = -V_i^2 B_{ii} + \sum_{j=1;j\neq i}^{N} V_i V_j G_{ij} \sin(\theta_i - \theta_j)$$
$$- \sum_{j=1;j\neq i}^{N} V_i V_j B_{ij} \cos(\theta_i - \theta_j) \tag{1}$$

where $\tilde{P}_i$ and $\tilde{Q}_i$ are the injected real and reactive power respectively of bus i. This means that in order to have the equations (1) well-defined for the buses of the external area, it is required that it has the voltage magnitudes V_i^{study} and phases θ_i^{study} that are adjacent to it as input signals. And having the external area output the voltage magnitudes V_i^{ext} and phases θ_i^{ext} will ensure that the study area has all the required bus variables available to make (1) well-defined for all its buses that are adjacent to the external area. The input and output signals are analogously defined for the study area.

Given this interface the external area can be linearized around a steady-state and reduced after which it is reconnected to the study area. The objective is to do the model reduction so that the dynamics of the nonlinear study area is affected as little as possible when replacing the nonlinear description of the external area with the reduced order linear model.

III. MODEL REDUCTION METHOD

Structured model order reduction is a model reduction technique that can be applied to systems composed of subsystems that are interconnected with some network dynamics. The idea is to reduce the model order of the subsystems while retaining the interconnection structure and keeping the global model error small. The general system (Fig. 2) to which structured model reduction can be applied is composed of q subsystems described by the transfer function matrix

$$G(s) = \begin{bmatrix} G_1(s) & & 0 \\ & \ddots & \\ 0 & & G_q(s) \end{bmatrix} =: \left[\begin{array}{c|c} A_G & B_G \\ \hline C_G & D_G \end{array} \right] \tag{2}$$

with transfer functions $G_k(s) \in \mathbb{C}^{p_k \times m_k}$ and an interconnecting network

$$N(s) = \left[\begin{array}{c|c} E(s) & F(s) \\ \hline H(s) & K(s) \end{array} \right] =: \left[\begin{array}{c|cc} A_N & B_{N,1} & B_{N,2} \\ \hline C_{N,1} & D_E & D_F \\ C_{N,2} & D_H & D_K \end{array} \right]. \tag{3}$$

We want to find a reduced order system that approximates the mapping from u_1^N to y_1^N defined by the lower linear fractional transformation

$$\mathcal{F}_l(N, G) = E(s) + F(s)(I - G(s)K(s))^{-1} G(s)H(s)$$
$$=: \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array} \right] \tag{4}$$
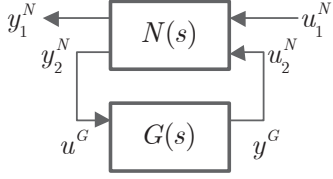
Fig. 2. The interconnected system. G is the system that should be reduced and N is the interconnecting network. u_1^N and y_1^N are external input and output respectively.

with the constraint that the interconnecting network N is retained, i.e. the objective is to find the reduced order system $\hat{G}$ such that

$$\|\mathcal{F}_l(N, G) - \mathcal{F}_l\left(N, \hat{G}\right)\|_\infty \qquad (5)$$

is made as small as possible and

$$\hat{G} \in \{F(s) : F(s) = \operatorname{diag}\{F_1(s), ..., F_q(s)\}\}$$

where $F_k(s) \in \mathbb{C}^{p_k \times m_k}$, $k = 1, ..., q$.

Finding the optimal minimum to (5) is very difficult, since it is a nonconvex optimization problem. We will therefore have to resort to suboptimal methods, which yield solutions satisfying the constraints while trying to minimize the norm of the model error. The model order reduction algorithm used in this paper is based on the idea of balanced truncation, see for example [22] and [27]. To enforce the structural constraints we use a generalization of balanced truncation as described in [28], [29], [25], [26]. The notation used here closely follows the one used in [25] and [26].

It uses the reachability and observability Gramians P and Q given by the Lyapunov equations

$$AP + PA^T + BB^T = 0, \qquad A^T Q + QA + C^T C = 0, \quad (6)$$

with the matrices A, B, C defined by (4). It is helpful to use a partition with the blocks Q_N, P_N for the interconnecting network that is not reduced and with the blocks Q_k, P_k for subsystem k that should be reduced separately, but in a way so that the closed-loop dynamics is retained.

$$Q = \begin{bmatrix} Q_N & Q_{NG} \\ Q_{NG}^T & Q_G \end{bmatrix}, Q_G = \begin{bmatrix} Q_1 & \cdots & Q_{1q} \\ \vdots & \ddots & \vdots \\ Q_{1q}^T & \cdots & Q_q \end{bmatrix} \quad (7)$$

$$P = \begin{bmatrix} P_N & P_{NG} \\ P_{NG}^T & P_G \end{bmatrix}, P_G = \begin{bmatrix} P_1 & \cdots & P_{1q} \\ \vdots & \ddots & \vdots \\ P_{1q}^T & \cdots & P_q \end{bmatrix} \quad (8)$$

The method balances the subsystems $G_k(s)$ by the coordinate transformation $x_k = T_k \bar{x}_k$ that makes the transformed Gramians $\bar{Q}_k = T_k^T Q_k T_k$ and $\bar{P}_k = T_k^{-1} P_k T_k^{-T}$ subsystem balanced, which means that

$$\bar{P}_k = \bar{Q}_k = \Sigma_k = \operatorname{diag}\{\sigma_{k,1}, ..., \sigma_{k,n_k}\},$$
$$\sigma_{k,1} \geq ... \geq \sigma_{k,n_k} \geq 0,$$
$$\sigma_{k,j} = \sqrt{\lambda_j(P_k Q_k)} = \sqrt{\lambda_j(\bar{P}_k \bar{Q}_k)}. \qquad (9)$$

Thus if the original state vector has the structure,

$$x = [x_N^T \ x_1^T \ ... \ x_q^T]^T, \qquad (10)$$

then the transformed system will have the states $\bar{x}$ defined by

$$T\bar{x} = x,$$

where

$$T = \operatorname{diag}(T_N, T_1, ..., T_q), \ T_N \in \mathbb{R}^{n_N \times n_N}, \ T_k \in \mathbb{R}^{n_k \times n_k},$$

and n_N and n_k are the order of system N and G_k respectively. A nonsingular matrix T defining the balancing coordinate transformation can always be found given that the system is stable, see Corollary 7.7 in [30].

Having made a coordinate transformation, either truncation or singular perturbation is used to calculate the reduced order systems $\hat{G}_i$ of the subsystems in G. To this end the structured Hankel singular values can guide the choice of which states to retain as explained below.

The strength of the structured model reduction algorithm is that the block-diagonal elements of the Gramians defined by (6) tell us how reachable and observable the states of the subsystems are when we control the global input signal u_1^N and observe the global output signal y_1^N (Fig. 2). More precisely, the following holds [25]:
If all states excepts the ones in subsystem k are zero at time zero, then

$$\|y_1^N(t)\|_{[0,\infty]}^2 = x_k(0)^T Q_k x_k(0). \qquad (11)$$

and assuming that all states of the interconnected system are zero at $t = -\infty$ and that we would like to control the states of subsystem k to the specific state $x_k(0) = x_k^*$, while the states $x_N(0)$ and $x_i(0)$, $i \neq k$ are free variables, then the minimum control signal satisfies

$$\min_{\substack{u_1^N \in L_2(-\infty, 0) \\ x(0) \in X_k^*}} \|u_1^N(t)\|_{[-\infty, 0]}^2 = (x_k^*)^T P_k^{-1} x_k^*,$$

$$X_k^* = \{x : x \text{ has structure (10) and } x_k = x_k^*\}.(12)$$

The structured Hankel singular values (9) together with (11) and (12) indicate which states that are most important to retain; the larger they are, the more controllable and observable will they be through the exogenous input u_1^N and output y_1^N. However if it is desired to have an upper bound on the model error one has to resort to LMIs [26].

IV. STRUCTURED MODEL REDUCTION OF POWER SYSTEMS

The structured model reduction algorithm accounted for in Section III is based on the notion of dividing the system into the subsystems N and G. This makes it suitable for application to power systems with one study area, which we want to retain a non-linear description of and one external area which we want to reduce. We will henceforth assume that the system G only consists of one subsystem that we want to reduce, i.e. with the notation introduced in Section III, $q = 1$. Although we make this assumption, there is

nothing preventing G from being composed of more than one subsystem, something that could be of interest in large power systems with several areas of particular interest. We now propose the following four-step algorithm:

1. Defining the model
A general power system will be described by differential algebraic equations (DAE) of the form

$$\dot{x} = f(x, x_{\mathrm{alg}}, u)$$
$$0 = g(x, x_{\mathrm{alg}}, u). \tag{13}$$

The states x will be describing the generators, controllers, etc., whereas the algebraic variables x_{alg} will be the voltages and phases of the buses as well as algebraic variables describing the generators, controllers, etc. The signal u will be an exogenous input to the power system. It could for instance describe time-varying loads. Divide this system into a study area, denoted N and an external area denoted G. A general system with this structure can be described by the DAEs

$$\dot{x}^G = f^G(x^G, x_{\mathrm{alg}}^G, u^G)$$
$$0 = g^G(x^G, x_{\mathrm{alg}}^G, u^G) \tag{14}$$

and

$$\dot{x}^N = f^N(x^N, x_{\mathrm{alg}}^N, u_1^N, u_2^N)$$
$$0 = g^N(x^N, x_{\mathrm{alg}}^N, u_1^N, u_2^N) \tag{15}$$

The variables u^G and u_2^N are the voltage magnitudes and phases of the buses at the tie-line as described in Section II and u_1^N is the same exogenous input as in (13), (Fig. 2).

2. Linearizing
In order to apply the structured model reduction algorithm described in Section III it is first necessary to linearize both the study area and the external area. By solving the power flow problem, the steady-state of the power system is acquired around which the linearization is done. The linearization of (14, 15) will take the form

$$\begin{pmatrix} \dot{x}^G \\ 0 \end{pmatrix} = \begin{pmatrix} A_{11}^G & A_{12}^G \\ A_{21}^G & A_{22}^G \end{pmatrix} \begin{pmatrix} x^G \\ x_{\mathrm{alg}}^G \end{pmatrix} + \begin{pmatrix} B_1^G \\ B_2^G \end{pmatrix} u^G$$

$$\begin{pmatrix} \dot{x}^N \\ 0 \end{pmatrix} = \begin{pmatrix} A_{11}^N & A_{12}^N \\ A_{21}^N & A_{22}^N \end{pmatrix} \begin{pmatrix} x^N \\ x_{\mathrm{alg}}^N \end{pmatrix}$$
$$+ \begin{pmatrix} B_{11}^N & B_{12}^N \\ B_{21}^N & B_{22}^N \end{pmatrix} \begin{pmatrix} u_1^N \\ u_2^N \end{pmatrix}.$$

What makes these systems easy to work with is that the algebraic variables x_G^{alg} and x_N^{alg} can be solved for

$$x_{\mathrm{alg}}^G = -A_{22}^{G^{-1}}(A_{21}^G x^G + B_2^G u^G)$$

$$x_{\mathrm{alg}}^N = -A_{22}^{N^{-1}}(A_{21}^N x^N + B_{21}^N u_1^N + B_{22}^N u_2^N).$$

If the matrices M^G and M^N select which algebraic variables the two subsystems output, i.e. the tie-line voltage magnitudes and phases, the DAEs can be recast into the following ordinary differential equations

$$\dot{x}^G = (A_{11}^G - A_{12}^G A_{22}^{G^{-1}} A_{21}^G)x^G + (B_1^G - A_{12}^G A_{22}^{G^{-1}} B_2^G)u^G$$
$$y^G = M^G\left(-A_{22}^{G^{-1}}(A_{21}^G x^G + B_2^G u^G)\right) \tag{16}$$

and

$$\dot{x}^N = (A_{11}^N - A_{12}^N A_{22}^{N^{-1}} A_{21}^N)x^N$$
$$+ \left[(B_{11}^N - A_{12}^N A_{22}^{N^{-1}} B_{21}) \quad (B_{12}^N - A_{12}^N A_{22}^{N^{-1}} B_{22})\right] \begin{pmatrix} u_1^N \\ u_2^N \end{pmatrix}$$

$$\begin{pmatrix} y_1^N \\ y_2^N \end{pmatrix} = -\begin{pmatrix} 0 \\ M^N \end{pmatrix} A_{22}^{N^{-1}}$$
$$\cdot (A_{21}^N x^N + B_{21}^N u_1^N + B_{22}^N u_2^N) + \begin{pmatrix} 0 & I \\ 0 & 0 \end{pmatrix} \begin{pmatrix} u_1^N \\ u_2^N \end{pmatrix} \tag{17}$$

We can note that the system N has one input signal u_1^N and one output signal y_1^N apart from the input/output pair that defines its interconnection with the external area G. These are the exogenous inputs and the global outputs and they will be elaborated upon in Subsection IV-A.

3. Model reduction
With G and N on the form (16) and (17) the state space equations for the interconnected system can readily be found for which the reachability and observability Gramians can be calculated with (6). Selecting the submatrices P_G and Q_G from the matrices (8) and (7) a change of coordinates for the system G can be found and guided by the structured Hankel singular values (9) the model order can be selected.

4. Nonlinear model
With the system G being reduced to

$$\hat{G}(s) = \left[\begin{array}{c|c} A_{\hat{G}} & B_{\hat{G}} \\ \hline C_{\hat{G}} & D_{\hat{G}} \end{array}\right]$$

it can be reconnect to the non-linear description of the study area yielding the reduced interconnected system

$$\dot{x}^{\hat{G}} = A_{\hat{G}} x^{\hat{G}} + B_{\hat{G}} u^{\hat{G}}$$
$$u_2^N = y_{\hat{G}} = C_{\hat{G}} x^{\hat{G}} + D_{\hat{G}} u^{\hat{G}}$$

$$\dot{x}^N = f^N(x^N, x_{\mathrm{alg}}^N, u_1^N, u_2^N)$$
$$0 = g^N(x^N, x_{\mathrm{alg}}^N, u_1^N, u_2^N)$$
$$u^{\hat{G}} = y_2^N = M_N x_{\mathrm{alg}}^N.$$

A. Algorithm preferences

Having linearized the system, we want to find the reduced system $\tilde{G}$ that makes (5) as small as possible. The outcome of the model reduction will of course be dependent on the choice of exogenous input signals u_1^N and output signals y_1^N. In the example that will be used to demonstrate the algorithm the input signals u_1^N were chosen as the loads of the buses 5 and 6 (Fig. 3). This choice could be of interest if one wants to design a power system stabilizer (PSS) at the generator connected to bus 1 that attenuates oscillations following load variations. However, the choice should be application dependent.

The most natural choice for the output variables y_1^N is to choose them as the voltage magnitudes and phases of the buses of the external area at the tie-lines. This makes a good choice considering how we interconnect the two areas. Assuming that the reduced order external system exhibited

the same voltage amplitudes and phases on the buses at the tie-lines as the full external system, then it would follow that the trajectories of the study system of both models would be identical, given that they had the same initial condition. Therefore if we can make the tie-line variables of the reduced system and the full system be close to each other, then the two study systems should have similar trajectories which is the objective of the model reduction.

B. Required assumptions

The structured model reduction algorithm, which is based on balanced truncation, require that the interconnected system is asymptotically stable, since the reachability and observability Gramians will not be well defined otherwise. A power system with only synchronous generators and loads will however always have an eigenvalue at the origin as long as no reference for the voltage phase angles is chosen. There are a number of different approaches that can be taken to accommodate for this. One way is to simply fix one of the generators to be the reference in a similar manner to the selection of a slack bus in the power flow problem [31].

Another possibility that may arise is that the power system in question contains an infinite bus that models an inert system which dynamics are unknown or simply disregarded. This could for instance be a neighboring country connected by a tie-line. If this is the case, the system will not have an eigenvalue at the origin and the Gramians are well defined.

A third option would be to choose a reference angle to remove the zero eigenvalue, do the model reduction and then revert the system to one without a reference. This procedure might however yield unstable dynamics since the reduced system will have an eigenvalue in the proximity of the origin, and if it is not being paid attention to, it might end up in the right half plane.

V. ALGORITHM DEMONSTRATION

To calculate the reduced order power system we have used the Power System Analysis Toolbox (PSAT) [32]. Its Simulink interface can be used to draw up the two subsystems N and G. After using the built-in method to solve the power flow problem, PSAT can return the two linearized subsystems.

Then to simulate the full system and the reduced order system at the presence of some perturbation, MATLAB's ode15s solver for stiff DAEs of index 1 was used. This particular solver was chosen because power systems with high-order generators and Automatic Voltage Regulators (AVR) are known to have both really fast and slow dynamics, i.e. they are stiff systems [33].

A. Model

The system that will be used to demonstrate the structured model reduction algorithm is the WSCC 3-machine, 9-bus system (Fig. 3) [34] with a connected infinite bus. We will choose the study area N to be defined by the generator connected to bus 1 and the buses 1, 4, 5 and 6, whereas the external area G is selected to be composed of the generators

connected to buses 2 and 3 respectively as well as the remaining buses. The synchronous machines are modeled as fourth order systems with connected AVRs of the standard IEEE model 1. The DAEs for each machine are

$$
\begin{aligned}
\dot{\delta} &= \Omega_b(\omega - 1) \\
\dot{\omega} &= (p_m - p_e - D(\omega - 1))/M \\
\dot{e}'_q &= \left(-e'_q - (x_d - x'_d)i_d + v_f\right)/T'_{d0} \\
\dot{e}'_d &= \left(-e'_d + (x_q - x'_q)i_q\right)/T'_{q0} \\
0 &= v_q + r_a i_q - e'_q + x'_d i_d \\
0 &= v_d + r_a i_d - e'_d - x'_q i_q \\
0 &= -v_d + v\sin(\delta - \theta) \\
0 &= -v_q + v\cos(\delta - \theta) \\
0 &= -p_e + (v_q + r_a i_q)i_q + (v_d + r_a i_d)i_d,
\end{aligned}
$$

where we recognize δ to be the synchronous machine internal angle and ω to be the angular velocity of the machine [31]. These equations hold assuming that the field saturation characteristic is linear and that the field voltage v_f has no feedback term depending on the rotor speed and active power produced by the machine. The AVRs are described by the DAEs

$$
\begin{aligned}
\dot{v}_m &= (v - v_m)/T_r \\
\dot{v}_{r1} &= \left(K_a\left(v_{ref} - v_m - v_{r2} - \frac{K_f}{T_f}v_f\right) - v_{r1}\right)/T_a \\
v_r &= \begin{cases} v_{r1} & \text{if } v_r^{min} \leq v_{r1} \leq v_r^{max} \\ v_r^{max} & \text{if } v_{r1} > v_r^{max} \\ v_r^{min} & \text{if } v_{r1} < v_r^{min} \end{cases} \\
\dot{v}_{r2} &= -\left(\frac{K_f}{T_f}v_f + v_{r2}\right)/T_f \\
\dot{v}_f &= -\left(v_f(K_e + S_e(v_f)) - v_r\right)/T_e
\end{aligned}
$$

with

$$
S_e(v_f) = A_e e^{B_e|v_f|}
$$

The different machines of the power system are connected through transmission lines. Each bus satisfies the algebraic equations (1) and thereby interconnect the generators.
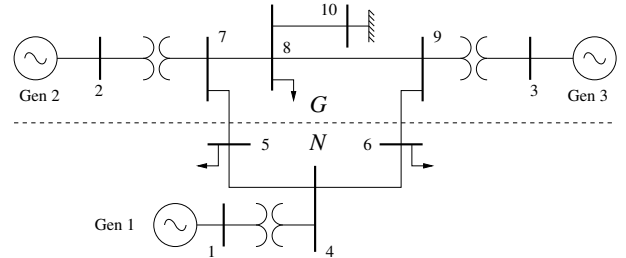

Fig. 3. The WSCC 3-machine, 9-bus system with an infinite bus.

B. Results

To evaluate the structured model order reduction algorithm it will be compared with ordinary balanced truncation applied to the external system G. We will introduce a perturbation in the internal generator angle δ of generator 1 to determine

the reduced order model's ability to capture the transients induced by the perturbation. This is important if one wishes to study how well damped the system is. A reduced order model with this quality could for instance be used to design a power system stabilizer (PSS) controller that dampens inter-area oscillations.

To decide what the model order of the reduced system should be, we can use the structured Hankel singular values (9). Although nothing conclusive can be inferred from them, they can be used for guidance. Fig. 4 shows that the size of the structured Hankel singular values is dropping quickly, indicating that there is a good chance the system can be modeled with only a few states. It turns out that at least three states are required to model the external area with satisfactory results.
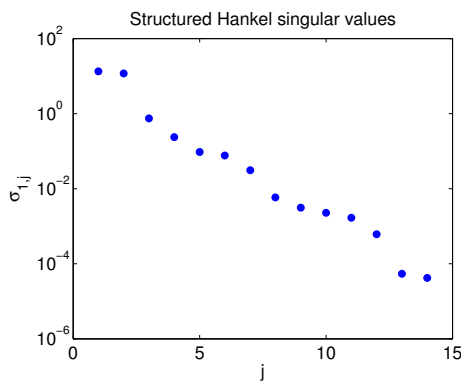

Fig. 4. The structured Hankel singular values of system G in a semilog scale.

The output variables y_1^N of the interconnected system that we try to model accurately are the voltage amplitudes and phases of the tie-lines buses. Fig. 5 and 6 show the transients of V_7 and θ_7 when the internal generator angle δ of generator 1 is perturbed 1.6% and the external area has been reduced from 16 states to 3. We see that the reduced model based on structured model reduction captures most of the interesting dynamics although it misses some in the higher frequencies. If ordinary balanced truncation is applied to reduce the external area to a system of order 3, then when interconnecting the reduced system $\hat{G}$ with the study area, we see that the system becomes unstable. Similar observations can be made for bus 9. To get an even better model approximation we can just add a few more states. Looking at Fig. 4 we see that there is a jump between the seventh and eighth structured Hankel singular value. This motivates adding four more states to the reduced model (Fig. 7, 8) to capture the high frequency dynamics that was lost in the reduced system with $\hat{G}$ of order 3. Doing this we see that the reduced model derived from the application of the structured model order reduction algorithm almost achieves a perfect fit with the full model whereas the application of ordinary balanced truncation yields a sizeable error.
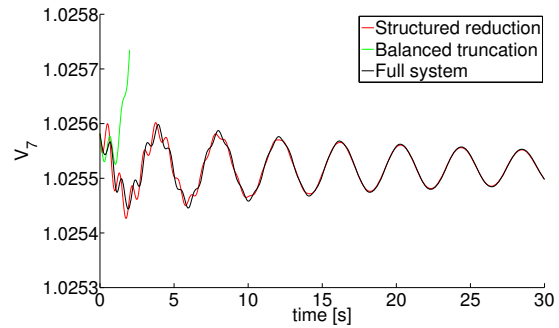

Fig. 5. Transients of V_7 at the tie-line bus with a third order system $\hat{G}$. Structured model reduction and ordinary balanced truncation are compared with the full system.
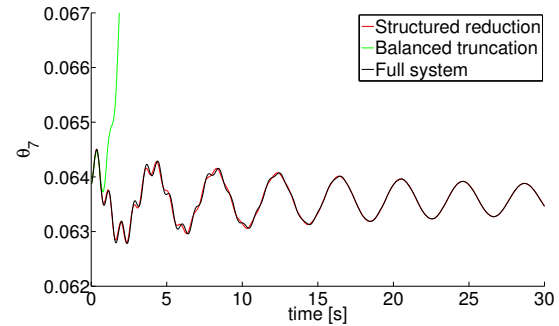

Fig. 6. Transients of θ_7 at the tie-line bus with a third order system $\hat{G}$. Structured model reduction and ordinary balanced truncation are compared with the full system.
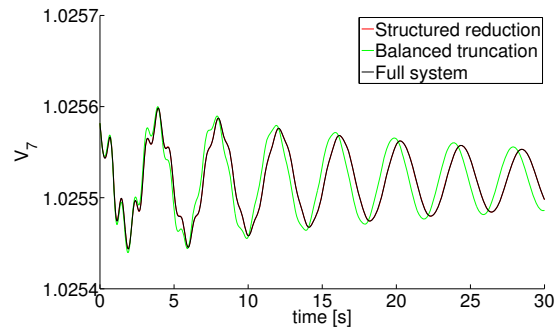

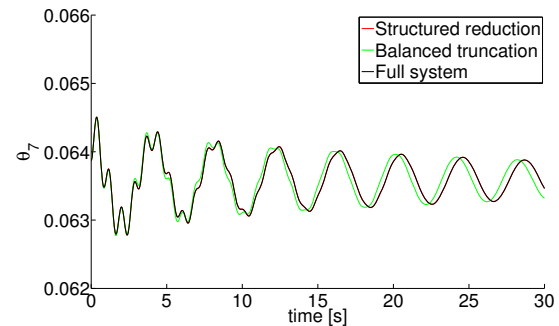Fig. 7. Transients of V_7 at the tie-line bus with a seventh order system $\hat{G}$.

Fig. 8. Transients of θ_7 at the tie-line bus with a seventh order system $\hat{G}$.

VI. Conclusion

We have proposed an algorithm for applying structured model order reduction to power systems divided into an external area that should be reduced and a study area. The resulting reduced order system is an interconnection of a reduced order linear system describing the external area and the nonlinear study area. The algorithm was demonstrated on the WSCC 3-machine, 9-bus system. It was found that only three states were required to describe the 16 states external system, when using the proposed algorithm. Ordinary balanced truncation was also applied to the external area to illustrate the importance of taking the full system into account when doing a local model reduction. In this case the reduction of the external area to a system of order three yielded an unstable system.

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