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Different approaches to model wind speed based on stochastic differential equations

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Motivation

- Use of wind speed models in the analysis of many aspects of power systems, e.g.,
  - Power system economics and operation
  - Generation capacity reliability evaluation
  - Dynamic studies and control of wind turbines
- Different types of models:
  - Time series
  - Four-component composite models
  - Kalman filters
  - Stochastic differential equations (SDEs)

Motivation

- Statistical characterization of the wind speed
  - Probability distribution: **Weibull**, Gamma, etc.
  - Autocorrelation: **Exponential**, Power-law, etc.
- Models used in the literature fail in reproducing one of the above characteristics
- Need of new models: **Stochastic Differential Equations**

Stochastic Differential Equations

- General form:

$$dx(t) = a(x(t),t) \cdot dt + b(x(t),t) \cdot dW(t)$$

  - $a(x(t),t)$ is the **drift** term
  - $b(x(t),t)$ is the **diffusion** term
  - $W(t)$ is a standard **Wiener process**

- Stationary processes:

$$a(x(t),t) \longrightarrow a(x(t))$$

$$b(x(t),t) \longrightarrow b(x(t))$$

Modeling Process

- Data analysis:
  - Probability distribution of the wind speed: $p_{\mathrm{D}}(x(t))$
  - Autocorrelation coefficient: α

- SDE construction: Find the drift and the diffusion of

$$dx(t) = a(x(t)) \cdot dt + b(x(t)) \cdot dW(t)$$

such that

$$p(x(t)) = p_{\mathrm{D}}(x(t))$$

$$r_x(\tau) = e^{-\alpha \cdot \tau}$$

Method I

- Ornstein-Uhlenbeck process

$$dy(t) = -\alpha \cdot y(t) \cdot dt + \sqrt{2 \cdot \alpha} \cdot dW(t)$$

$$p(y(t)) = \mathcal{N}(0,1)$$

$$r_y(\tau) = e^{-\alpha \cdot \tau}$$

Method I

- Memoryless transformation

$$x(t) = F_D^{-1}\left(\Phi(y(t))\right)$$

- Itô formula gives

$$dx(t) = a_{\mathrm{D}}(x(t)) \cdot dt + b_{\mathrm{D}}(x(t)) \cdot dW(t)$$

$$p(x(t)) = p_{\mathrm{D}}(x(t))$$

$$r_x(\tau) \leq e^{-\alpha \cdot \tau}$$

Method I

- Can be applied for every continuous probability distribution
- Leads to very complicated SDEs
- Due to the application of the memoryless transformation the **autocorrelation is not guaranteed**

Fokker-Planck Equation

- Gives the time evolution of the probability densities of stochastic processes represented by SDEs

- Stationary state:

$$0 = -a(x(t)) \cdot p(x(t)) + \frac{1}{2} \cdot \frac{\partial}{\partial x(t)} \left[b^2(x(t)) \cdot p(x(t)) \right]$$

Fokker-Planck Equation

- Solved for the diffusion term:

$$b^2(x(t)) = \frac{2}{p_{\mathrm{D}}(x(t))} \cdot \int_{-\infty}^{x(t)} a(z(t)) \cdot p_{\mathrm{D}}(z(t)) \cdot dz(t)$$

- Solved for the drift term:

$$a(x(t)) = b(x(t)) \cdot \frac{\partial b(x(t))}{\partial x(t)} + \frac{1}{2} \cdot b^2(x(t)) \cdot \frac{\partial \ln p_{\mathrm{D}}(x(t))}{\partial x(t)}$$

Method II

- Find the formulation of the drift term $a(x(t))$ to guarantee exponential autocorrelation

- The diffusion term is obtained by solving:

$$b^2(x(t)) = \frac{2}{p_{\mathrm{D}}(x(t))} \cdot \int_{-\infty}^{x(t)} a(z(t)) \cdot p_{\mathrm{D}}(z(t)) \cdot dz(t)$$

Method II

- Regression theorem:

$$\frac{dE[x(t)]}{dt} = -\alpha \cdot E[x(t)] \longrightarrow r_x(\tau) = e^{-\alpha \cdot \tau}$$

- Sufficient condition for having exponential autocorrelation:

$$a(x(t)) = -\alpha \cdot (x(t) - \mu_D)$$

Method II

- Can be applied for every continuous probability distribution
- **Exact exponential autocorrelation**

Method III

- Find the formulation of the diffusion term $b(x(t))$ to guarantee exponential autocorrelation
- The drift term is obtained by solving:

$$a(x(t)) = b(x(t)) \cdot \frac{\partial b(x(t))}{\partial x(t)} + \frac{1}{2} \cdot b^2(x(t)) \cdot \frac{\partial \ln p_{\mathrm{D}}(x(t))}{\partial x(t)}$$

Method III

- We haven't found a theoretically justified way to define the diffusion term, but

$$b(x(t)) = \sqrt{2 \cdot \alpha} \cdot \sigma_{\mathrm{D}}$$

  has been used for Gaussian-related distributions

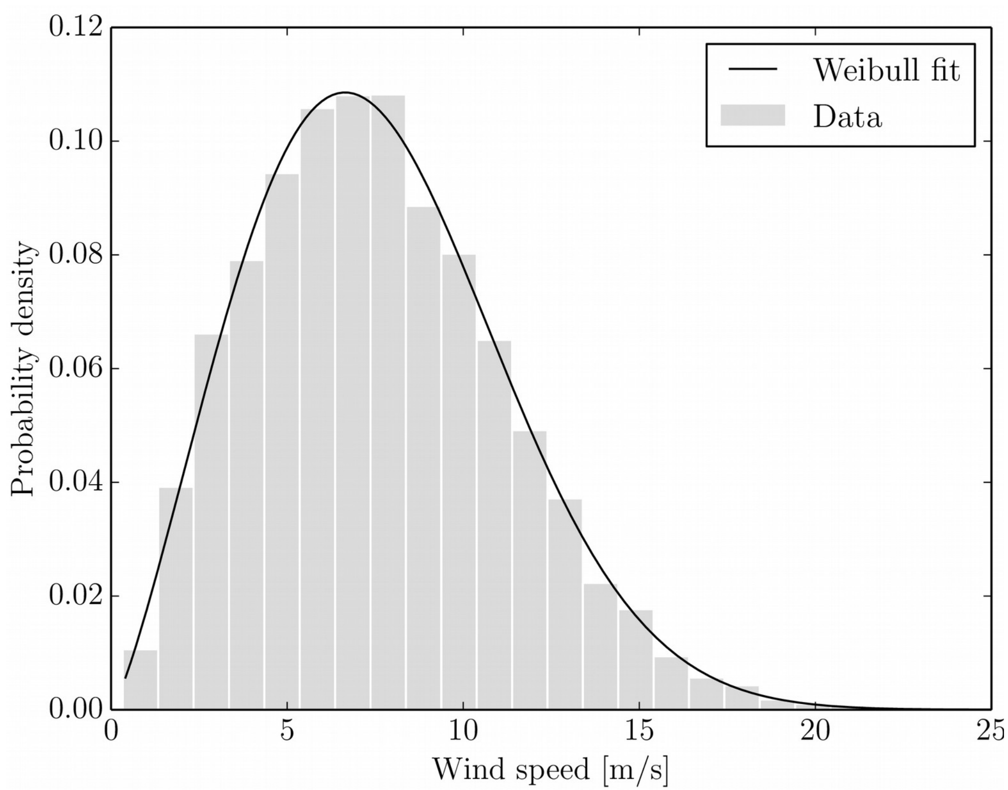
- Let's try it for non-Gaussian distributions!

Method III

- Can be applied for every continuous probability distribution
- According to simulations: **Approximated exponential autocorrelation**
- Appearance of numerical inconsistencies: e.g. negative wind speed values

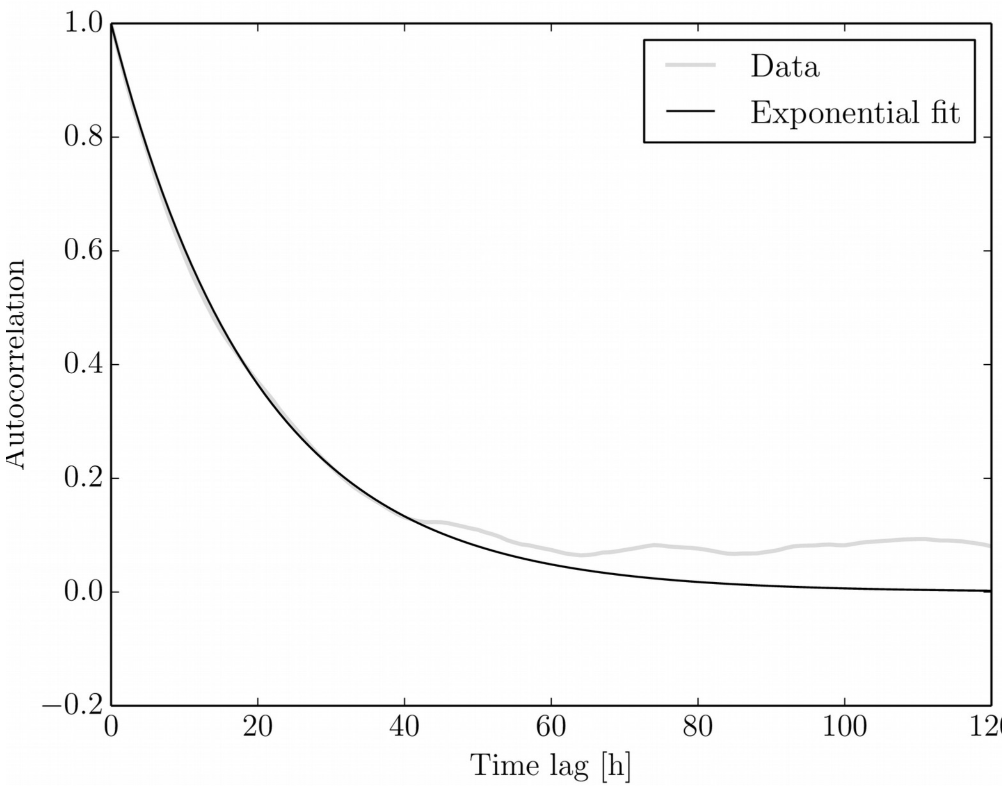
Case Study

- Data analysis: Probability distribution



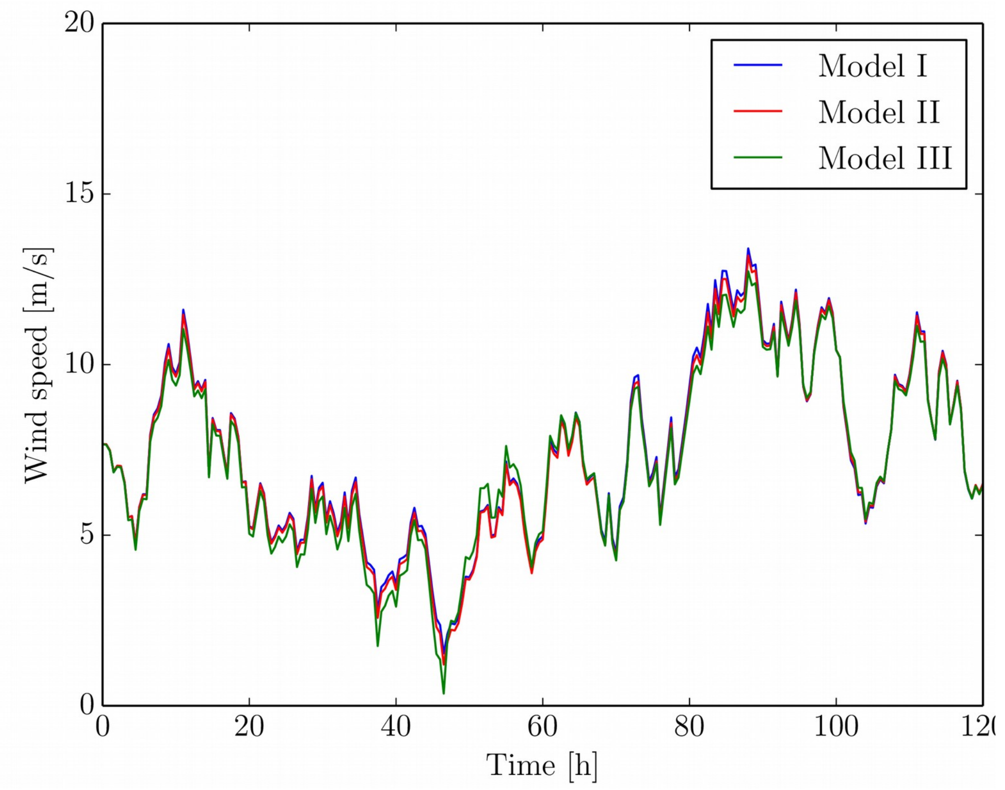
Case Study

- Data analysis: Autocorrelation



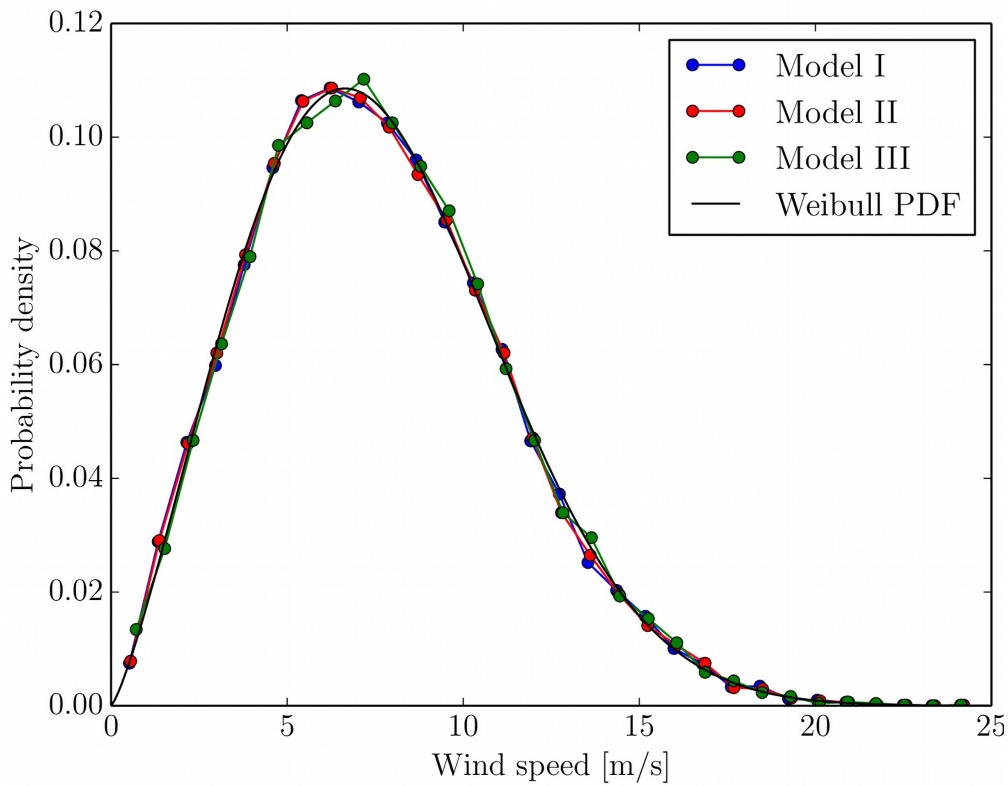
Case Study

- Comparison of trajectories



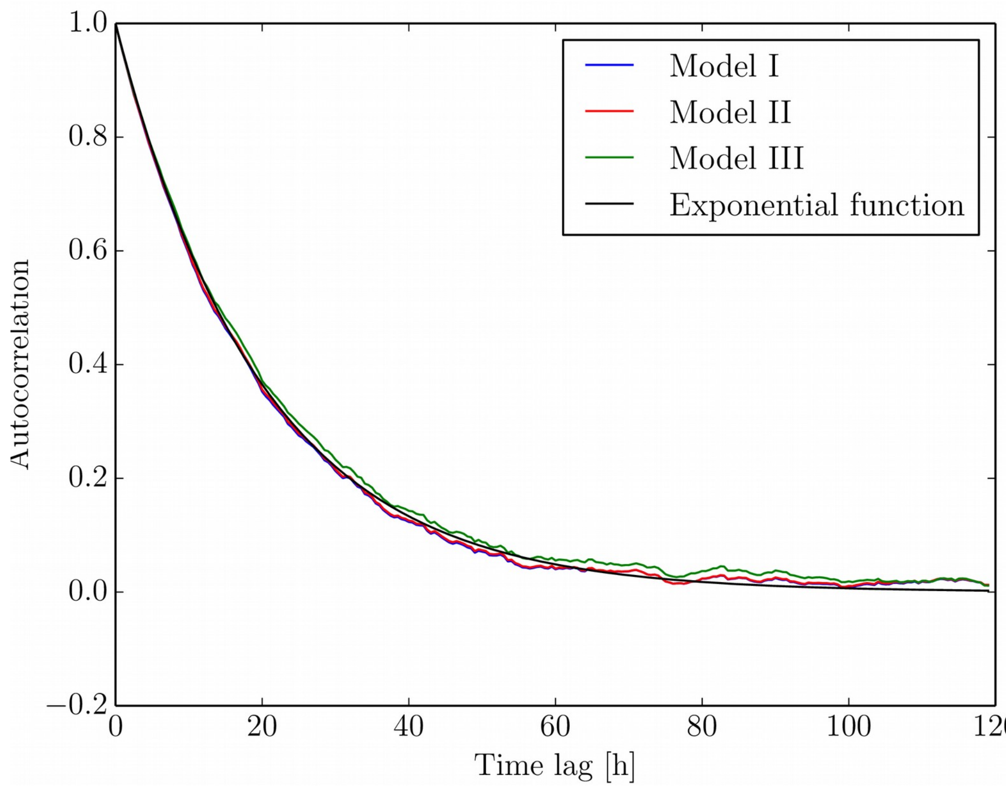
Comparison

- Comparison of probability distribution



Case Study

- Comparison of autocorrelation



Conclusions

- SDEs are a promising tool to model stochastic phenomena in power systems, e.g. wind speed
- Only Method II guarantees both the probability distribution and the exponential autocorrelation
- Methods I and III give the correct probability distribution but just an approximation to the exponential autocorrelation
- Method III leads to models that can produce trajectories with numerical inconsistencies: e.g. negative wind speed values

References

- **Method I**:

  R. Zárate-Miñano, M. Anghel, and F. Milano, "Continuous wind speed models based on stochastic differential equations", *Applied Energy*, 104, 42-49, April 2013.

- **Method II**:

  R. Zárate-Miñano, F. Milano "Construction of SDE-based wind speed models with exponentially decaying autocorrelation", *Renewable Energy*, 94, 186-196, August 2016.

References

- **Method III (for Gaussian-related distributions)**:

R. Calif, “PDF models and synthetic model for the wind speed fluctuations based on the resolution of Langevin equation”, *Applied Energy*, 99, 173-182, November 2012.

- **Comparison of methods**:

R. Zárate-Miñano, F. M. Mele, and F. Milano “SDE-based wind speed models with Weibull distribution and exponential autocorrelation”, *2016 IEEE PES General Meeting*.

Thank you!